Do Endogenous Shocks Matter?

Job Market Paper

Pavel Kapinos

First draft: December 9, 2003

Current draft: November 5, 2004

Abstract

The closed economy New Keynesian literature has given considerable attention to endogenous shocks to the baseline Phillips curve that arise due to the cost channel of transmission of monetary policy or sticky nominal wage contracts. Either of these two endogenous shocks is capable of producing a meaningful tradeoff between the output gap and price inflation in response to a technological shock. However, the cost channel plays an important role in response to that shock only when the nominal wages are assumed to be flexible. Once they are sufficiently sticky, the cost channel's presence has negligible impact. To be able to produce the so-called 'price puzzle'—the positive response of prices to a contractionary monetary policy shock, the New Keynesian model has to incorporate all three nominal frictions: sticky prices, sticky wages, and the cost channel. Comparison of an interest rate rule that targets expected future inflation to a money supply rule that implies targeting the current price level suggests that the latter dramatically reduces the price inflation variance but, depending on the theoretical setup, is also likely to raise the volatility of the output gap.

1 Introduction

The traditional New Keynesian Phillips curve posits current inflation as a function of expected inflation in the next period and the current output gap, which, in stark contrast to empirical facts, implies that disinflation can be carried out costlessly. Researchers have become increasingly dissatisfied with this so-called canonical specification of inflation dynamics described in Clarida, Galí, and Gertler (1999), where the Phillips curve is of the form:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t, \tag{1}$$

(where π is inflation and $\tilde{y}$ is the output gap). The most popular way of addressing this problem has been to append an ad hoc stochastic cost-push shock to this New Keynesian Phillips curve, which may not be a satisfactory solution, because the tradeoff between the output gap and inflation should naturally arise from the model's setup. This paper discusses two mechanisms that give rise to such endogenous shocks: the cost channel, whereby the nominal interest rate enters a firm's marginal cost function, and the nominal wage stickiness, whereby households bargain for the nominal wage contracts that may not be reset for some time. The theoretical implications of these two endogenous shocks were first explored in Ravenna and Walsh (2002) and Erceg, Henderson, and Levin (2000), respectively.

An attractive feature of these shocks is that they introduce a tradeoff between the output gap and inflation by means of propagating the technological shock. In so doing, there is no need to introduce additional, possibly ad hoc, shocks to explain this tradeoff. Furthermore, the presence of endogenous shocks is likely to reduce the importance of other, ad hoc shocks. For instance, in a recent paper, Ireland (2004) argues that in the post-1980 time period ad hoc cost-push shocks account for most of the variability in inflation, nominal interest rate, and output gap at long horizons, whereas the technological shock contributes remarkably little. In the case where the cost channel is present or nominal wages are sticky, part of what is assumed to be a cost-push shock arises endogenously from a technological shock. Therefore, the role of the technological shock is likely to be vastly underestimated and that of the cost-push shock overestimated.

The main focus of this paper is to gauge the relative importance of the cost channel and nominal wage stickiness. This investigation is carried out in two directions. First, the cost channel's role in response to a technological shock is contrasted with the relative importance of sticky nominal wages. Simulation results suggest that once nominal wages are assumed to be sticky, the cost channel adds very little to the model's dynamics in response to a technological shock. Second, the cost channel may play an important role in resolving the so-called 'price puzzle'—a positive impulse response in prices to a contractionary monetary policy shock. In a model with sticky prices and nominal wages, the presence of the cost channel produces price impulse responses that are found in the empirical literature.

These theoretical impulse responses are obtained after estimating an empirical monetary policy rule that is different from the one derived under the assumption of flexible nominal wages and that indicates that the Federal Reserve may not react to inflation as aggressively as previously thought. A formal derivation of the monetary policy rule as a second-order approximation of the utility function suggests that, in addition to measures of inflation and output gap, the central bank should also target a measure of wage inflation, which has been absent from the empirical estimates so far. This paper fills this gap in the literature.

Recent theoretical work has also resurrected the debate between interest rate and money supply rules. In particular, while the former implies that nominal interest should respond to an expected inflation term, the latter indicates that it should target the current price level. Ball, Mankiw, and Reis (2003) have found the price targeting rule to be optimal in a model with sticky information. This paper presents an estimate of a money demand equation from the US data and uses it in the computer simulation exercise to compare it to the Taylor-like rule that targets expected inflation. Although more work needs to be done in this direction, preliminary results indicate that price targeting results in less welfare loss, just as Ball, Mankiw, and Reis (2003) claim.

The rest of the paper is organized as follows. Section 2 presents a New Keynesian model with sticky nominal wages and the cost channel. Section 3 discusses the theoretical underpinnings of an optimal monetary policy rule in this setting and provides its empirical estimates using recent United States data. Section 4 uses a Matlab implementation of an efficient algorithm for solving systems of rational expectations equations described in Anderson and Moore (1985)[1] to simulate the model's response to a positive technological shock and a contractionary monetary policy shock. Section 5 contrasts alternative conducts of monetary policy, namely, an interest rate rule versus a money supply rule. Finally, Section 6 concludes.

2 The Model

The model has three sectors: households, producers, and the monetary authority (or the central bank). The nominal wage stickiness arises from the fact that households face an exogenous probability of not being able to renegotiate their wages at any given time period. Similarly, sticky prices are the product of the producers' facing an exogenous probability of not being able to reset their prices. The cost channel arises from the fact that a firm's marginal cost is a function of the nominal interest rate. Finally, the monetary authority follows a rule that is meant to maximize an approximation of the household utility function under discretion. The model's complete setup is detailed below.

2.1 Households and Nominal Wage Stickiness

The economy is populated by homogeneous agents, i, who derive utility from consumption of a composite good C_t and disutility from providing labor. A representative household solves the utility maximization problem:

$$\max_{\{C_t, N_t, D_{t+1}\}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N(i)_t^{1+\phi}}{1+\phi} \right\} \right] \tag{2}$$

subject to the following budget constraint:

$$\int_0^1 P_t(i) C_t(i) di + R_t^{-1} D_{t+1} \leq D_t + W(i)_t N(i)_t, \tag{3}$$

where C_t, N_t, R_t, D_t are consumption, labor supply, gross nominal interest rate, and debt holdings at time t, respectively. Since goods are differentiated, the pricing index is given by:

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon_p} di \right)^{\frac{1}{1-\epsilon_p}}, \tag{4}$$

[1] The method for solving such systems was originally proposed by Blanchard and Kahn (1980).

where $\epsilon_p > 1$ is the elasticity of substitution between different goods. Households' expenditure minimization problem allows for the derivation of the demand function for a differentiated good:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} C_t. \tag{5}$$

Because consumption is symmetric across households, $\int_0^1 P_t(i)C_t(i)di = P_t C_t$.

Supply of labor, on the other hand, is not symmetric but differentiated. The wage index is given by:

$$W_t = \left(\int_0^1 W_t(i)^{1-\epsilon_w} di\right)^{\frac{1}{1-\epsilon_w}}. \tag{6}$$

Because producers minimize their expenditure on labor, each household faces the following demand curve for its differentiated labor supply:

$$N_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\epsilon_w} N_t. \tag{7}$$

The probability that a household may not be able to reset the nominal wage that it receives for its labor input is θ_w. Therefore, when households renegotiate their wage and maximize their utility subject to the budget constraint and demand for their labor, they pick a $W_t^*(i)$ that satisfies the following first-order condition:[2]

$$E_t\{\sum_{s=0}^{\infty}(\theta_w\beta)^s \left[\frac{W_t^*(i)}{P_{t+s}} - MRS_{t,t+s}(i)\right] N_{t+s}(i)\} = 0, \tag{8}$$

where $MRS_{t,t+s}(i) = -\frac{\partial U_t(i)/\partial N_t(i)}{\partial U_t(i)/\partial C_t}$. Notation on this term reflects the fact that the household's nominal wage in periods $t+s$ is going to be fixed at the level established in period t. Log-linearizing this expression and rearranging gives:

$$E_t\{\sum_{s=0}^{\infty}(\theta_w\beta)^s \left[(1+\epsilon_w\phi)w_t^* + \zeta_{t+s} - mrs_{t+s} - (1+\epsilon_w\phi)\sum_{\tau=t+1}^{s}\xi_\tau]\right]\} = 0, \tag{9}$$

where ξ is the wage inflation and ζ is the real wage defined as

$$\zeta_t = \log\left(\frac{W_t}{P_t}\right) = \zeta_{t-1} + \xi_t - \pi_t. \tag{10}$$

[2]Note that I am ignoring the constant markups on a differentiated products or labor, as well as efficiency subsidies to the firms, since they would drop out after log-linearization and would not affect simulation results. Also, if households or firms are unable to reset wages or prices, their wage or price is indexed by the steady state level of inflation, $\Pi = 1$. As Erceg et al. (2000) point out, this leads to the steady state being the same as if wages and prices were fully flexible.

Quasi-differencing and making use of the log-linearized definition of the wage index, we obtain an equation that describes the wage inflation dynamics:

$$\xi_t = \lambda_w [mrs_t - \zeta_t] + \beta E_t \xi_{t+1}, \tag{11}$$

where $\lambda_w = \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\epsilon_w\phi)}$.

The other first-order conditions imply that:

$$\beta R_t E_t \left(\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right) = 1, \tag{12}$$

which is the standard Euler equation, and

$$MRS_t = C_t^{\sigma} N_t^{\phi}. \tag{13}$$

The latter condition emphasizes that when nominal wages are sticky, the marginal rate of substitution between consumption and labor may not be equal to the real wage. (This gap is also reflected in equation (11) describing the wage inflation dynamics.) It is only if nominal wages are perfectly flexible, i.e., $\theta_w = 0$, that $MRS_t = \zeta_t$.

Finally, after aggregating, imposing the market clearing condition $C_t = Y_t$, and log-linearizing (12), one can derive the aggregate demand equation in terms of output:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(r_t - E_t\{\pi_{t+1}\}), \tag{14}$$

where $r_t = R_t - 1$ is the nominal interest rate.

2.2 Firms and the Cost Channel

Firms are monopolistically competitive and produce differentiated goods using the following production function:

$$Y_t(f) = Z_t N_t(f), \tag{15}$$

where the logarithm of the technological shock follows an AR(1) process:

$$z_t = \rho z_{t-1} + \varepsilon_t. \tag{16}$$

The presence of the cost channel is modeled by requiring firms to borrow cash (from a financial intermediary whose role in the model is otherwise unimportant) for one period to pay out wages to the laborers. This is, the firm pays wages at the beginning of the period with borrowed cash and returns the borrowing at the end of the period after the output has been produced. If there is no need for firms to borrow cash, the cost channel is absent. The real marginal costs, therefore, are given by:

$$MC_t = \frac{W_t R_t^{\chi}}{P_t Z_t}, \tag{17}$$

where χ is the share of wage payments that have to be financed by borrowing. $\chi = 0$ indicates that the cost channel is absent, and $\chi = 1$ that it is present to the full extent.[3] Note that the definition of the firm's marginal cost implies that if the nominal wages are perfectly flexible, they will move exactly in the opposite direction from the nominal interest rates, thus offsetting some of the volatility associated with the cost channel. Therefore, introducing nominal wage stickiness should augment the effect of the cost channel.[4]

As is well known, inflation dynamics in the New Keynesian models with the Calvo-pricing mechanism are given by the following Phillips curve:[5]

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \lambda_p mc_t, \tag{18}$$

where mc_t is the deviation of the marginal cost from the non-distortionary steady state of zero, $\lambda_p = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}$, and θ_p is the probability that a firm will keep the price of its product fixed for one period. For quarterly data, θ_p is usually set to 0.75, so that prices may fully adjust over the span of one year. As

[3] In Ravenna and Walsh (2002), $\chi = \{0, 1\}$ is an indicator variable but in this paper it is used as a parameter characterizing the extent of cost channel's presence or the share of wage payments that have to be financed by credit. Alternatively, as Rabanal (2003) points out, because in estimation this parameter is sometimes estimated to be greater than 1, χ can be interpreted as the share of firms that finance on credit times the average markup over the federal funds rate that they pay to obtain credit. In the present theoretical setup, χ is constrained between 0 and 1.

[4] Rabanal (2003) stresses that the assumption of price and nominal wage stickiness leads to sticky *real* wages.

[5] The original formulation of this mechanism is due to Calvo (1983). Yun (1996) incorporated it into a discrete-time model, variants of which are used throughout the literature. For an explicit derivation of the Phillips curve, which is similar to the derivation of the equation describing the wage inflation dynamics, see Woodford (2003) or Sbordone (2002).

will be demonstrated later, the presence of either the cost channel or nominal wage stickiness gives rise to an endogenous shock to the Phillips curve in the form of the nominal interest rate. This breaks down the canonical representation and produces a meaningful tradeoff between the output gap and inflation.

2.3 Flexible-price and Natural Equilibria

Distortions due to sticky prices and nominal wages may lead to suboptimal allocations. The extent of these distortions can be measured by comparing them to the values of variables in an economy where these distortions are eliminated. In the discussion below, all 'bar' variables designate their flexible-price level.

From the households' first-order condition that describes the consumption-leisure tradeoff and using the log-linearized form of the production function, we get:

$$\bar{\zeta}_t = (\sigma + \phi)\bar{y}_t - \phi z_t. \tag{19}$$

Equating the marginal cost of labor (including the cost of borrowing, if the cost channel is present) with the marginal product of labor, we have:

$$\bar{\zeta}_t = z_t - \chi \bar{r}_t. \tag{20}$$

Next, we can solve for the flexible-price level of output:

$$\bar{y}_t = \frac{1+\phi}{\sigma+\phi} z_t - \frac{\chi}{\sigma+\phi} \bar{r}_t. \tag{21}$$

Finally, from the Euler equation, we obtain the flexible-price interest rate as:

$$\bar{r}_t = \sigma(E_t \bar{y}_{t+1} - \bar{y}_t). \tag{22}$$

Note that unlike in Erceg et al. (2000), the flexible price equilibrium is not Pareto-optimal, because of the distortion associated with the cost channel. If this last distortion were eliminated by setting $\chi = 0$, we would obtain the Pareto-optimal, or 'natural', equilibrium described by barred variables with the superscript 'n', given by the following:

$$\bar{\zeta}_t^n = z_t, \tag{23}$$

$$\bar{y}_t^n = \frac{1+\phi}{\sigma+\phi} z_t, \quad (24)$$

and

$$\bar{r}_t^n = \sigma(E_t \bar{y}_{t+1}^n - \bar{y}_t^n). \quad (25)$$

It is against these 'natural' values that the actual levels of variables are going to be compared for the purpose of welfare analysis.

2.4 Sticky Price Equilibrium

Because there are three distortions present in this economy—from fixed prices, fixed nominal wages, and the cost channel—the actual levels of variables are going to be different from the natural ones. The variables that describe 'gaps' between actual and natural values are marked by '$\tilde{}$', i.e., for a variable x, $\tilde{x} = x - \bar{x}^n$. In welfare terms, these gaps describe the social losses that arise due to nominal frictions.

On the producer side of the model, the log marginal cost can be defined as:

$$mc_t = \zeta_t + \chi r_t - z_t. \quad (26)$$

Using the definition of the natural real wage (23), the Phillips curve (18), and the fact that the marginal rate of substitution can be expressed in terms of the output gap, i.e., $mrs_t = (\sigma + \phi)\tilde{y}_t + \bar{\zeta}_t^n$, inflationary dynamics can be described by the following equation:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_p(\sigma + \phi)\tilde{y}_t + \lambda_p(\zeta - mrs_t) + \chi\lambda_p r_t. \quad (27)$$

The monetary authority can no longer set both the output gap and inflation to zero, because of the presence of two endogenous shocks: the nominal interest rate, which arises due to the cost channel, and the gap between the real wage and the marginal rate of substitution between labor and consumption, which arises due to nominal wage stickiness. Note that it is only the presence of one of these shocks that is sufficient to break down the canonical representation.

On the consumer side, one can rewrite the aggregate demand relation (14) in terms of the output gap:

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}(\tilde{r}_t - E_t\{\pi_{t+1}\}). \quad (28)$$

The other structural equations in this model are the definitions of the natural levels of output and interest rate and their gaps, definitions of the marginal rate of substitution and the real wage rate (10), as well as the wage inflation equation (11). Disturbances to the system are produced by stochastic technological shocks whose law of motion is described by (16) or by monetary policy shocks discussed below. Finally, a monetary policy rule closes the system.

3 Monetary Policy Rule

The Appendix briefly details that taking the second-order approximation of the representative agent's utility function produces an objective function in terms of the output gap, price inflation, and wage inflation, where the latter term arises due to sticky nominal wages. Maximizing this function subject to the Phillips curve, aggregate demand, and the definition of the real wages, under discretion one can derive the following interest-rate rule:

$$r_t^* = \gamma_{\tilde{y}} \tilde{y}_t + \gamma_\pi E_t \pi_{t+1} + \gamma_\xi E_t \xi_{t+1}, \tag{29}$$

where γ_ξ is set to 0 when the nominal wages are assumed to be flexible. Following Clarida, Galí, and Gertler (1999) who implicitly assume flexible nominal wages and set $\gamma_\xi = 0$, I adjust for the persistence of nominal interest rates found in the data and reformulate the rule as:

$$r_t = (1 - \rho)\{\gamma_{\tilde{y}} \tilde{y}_t + \gamma_\pi E_t \pi_{t+1} + \gamma_\xi E_t \xi_{t+1}\} + \rho r_{t-1}, \tag{30}$$

where the last term allows for the persistence in interest rates.

To estimate this monetary policy rule, I used the generalized method of moments with the quarterly sample from 1983:I to 2003:IV.[6] The nominal interest rate in the policy rule is represented by the federal funds rate (FFR), output gap by quadratically detrended GDP, so as to make comparisons to Clarida, Galí

[6] All data, except nominal wages, are time series (1983.1–2003.12) from the FRED II database maintained by the Saint Louis Federal Reserve (http://research.stlouisfed.org/fred2/). Nominal wages are the private industries series from the Bureau of Economic Analysis Table 2.7 (A and B). The measure of money used in Section 5 is given by M2.

and Gertler (1999) more immediate,[7] price inflation by percentage changes in the consumer price index (CPI), and wage inflation by percentage changes in the nominal wage. The set of instruments included 3 lags of the output gap, price inflation, wage inflation, and the 3-month treasury bill rate.

Table 1: Estimates of the Interest Rate Rule (30)

	Parameters				Test
	γ_π	γ_ξ	$\gamma_{\tilde{y}}$	ρ	J
$\theta_w, \theta_p > 0$	1.481	1.095	1.046	0.758	5.302
	(0.220)	(0.369)	(0.214)	(0.056)	(0.723)
$\theta_w = 0$	2.082		1.218	0.828	6.806
	(0.189)		(0.357)	(0.054)	(0.657)
$\theta_p = 0$		2.287	2.535	0.947	9.074
		(0.729)	(0.862)	(0.018)	(0.431)
CGG	2.15		0.93	0.79	
	(0.40)		(0.42)	(0.04)	

Estimates of the three versions of the interest rate rule are presented in Table 1. In neither case does the p-value for the J-statistic reject the overidentifying restrictions implied by the instrumental variables. In the specification where nominal wages are assumed to be sticky, all coefficients are significant, with the values on the expected price and wage inflation terms greater than 1, which implies that the rule is stabilizing. Interestingly, unlike in Clarida et al (1999)[8] , the coefficient on the output gap term is also greater than one and significant, which describes the Fed's stance vis-à-vis this variable as aggressive. Estimates of the rule where wages are assumed to be flexible yield results that are very similar to Clarida et al (1999). Assuming

[7]Some researchers consider detrended GDP a poorer measure of the output gap. See Favero's (2001) Chapter 7 for an illustration. Alternative results with H-P filtered output used as a measure for the output gap are also presented in the Appendix. These results lead to qualitatively similar conclusions.

[8]Note that there are some differences between these results and the ones from that study in the sample and instruments used. Hence one should expected the results to be somewhat different.

sticky wages and flexible prices produces logically consistent results, although the standard errors on the estimates of γ_ξ and $\gamma_{\tilde{y}}$ increase several times. Hence it appears that assuming sticky prices is critical to the estimation of a monetary policy rule. Either of these rules is going to be used in the model's simulations conducted in the next section, and the choice of a rule will depend on assumptions about the value of θ_w and θ_p. It is important to note, however, that once one assumes sticky nominal wages and the Fed's concern with wage inflation that follows from this assumption, the extent of aggressiveness vis-à-vis price inflation decreases substantially.

4 Model's Parametrization

Several empirical studies have recently attempted to estimate the structural parameters of the New Keyensian model using various methodological techniques. For instance, Amato and Laubach (2000) minimize the distance between theoretical and empirical impulse responses within a VAR framework, Rabanal and Rubio-Ramírez (2003) use Bayesian maximum likelihood, and Kapinos (2004a) employs the generalized method of moments. Because the empirical systems do not have enough time series moments to estimate all of the structural parameters freely, additional restrictions need to be imposed.[9] Different approaches produce fairly different results in the main parameters: the duration of price contracts is between 2.7 and 6.2 quarters[10]; the duration of nominal wage contracts is between 2.1 and 4.0[11]; the inverse intertemporal elasticity of substitution, σ, is between 0.26 and 4.55; and the inverse labor supply elasticity, ϕ is between 0.1 and 1.68. Other studies that assume somewhat different model specifications produce results that are comparable to these ranges.

This lack of certitude in empirical estimates has led theoretical researchers who focus on simulation exercises to adapt parametric conventions.[12] For instance, the discount factor, β, is almost invariably set to

[9]See Kapinos (2004a) for a detailed discussion.

[10]This is, θ_p may vary between 0.63 and 0.84.

[11]This is, θ_w may vary between 0.52 and 0.75.

[12]Erceg, Henderson, and Levine (2000) and Ravenna and Walsh (2002), for example, use parameter values that are practically

0.99 (although empirical studies frequently produce lower estimates); the utility function is assumed to be nearly logarithmic in both consumption and labor, hence both σ and ϕ are set to values that are close to 1; and both price and wage contracts are assumed to last for a year, hence $\theta_w = \theta_p = 0.75$. These conventions are adopted here as well and the model's parameters are listed in Table A in the Appendix.

5 Theoretical Impulse Responses: Does the Cost Channel Matter?

This section considers the theoretical impulse response to two shocks: technological and monetary policy. The cost channel does not seem to play a quantitatively important role in response to the technological shock; its qualitative importance is also eroded once nominal wages are assumed to be sticky. In response to a contractionary monetary policy shock, however, it presence is necessary to produce the positive response in prices—a phenomenon that has been called the 'price puzzle' in the empirical literature.

5.1 Impulse Responses to a Positive Technological Shock

In a setup with flexible nominal wages, Ravenna and Walsh (2002) consider a theoretical leaning-against-the-wind rule, which under discretion allows the monetary authority to set both the output gap and price inflation to zero when the cost channel is absent. They then show that once the cost channel is present, there arises a meaningful tradeoff between the output gap and inflation. However, replacing this theoretical discretionary rule with an empirical rule discussed above eliminates much of the cost channel's impact. Furthermore, its presence seems to matter remarkably little in several theoretical settings.

the same as in this paper.

5.1.1 Sticky-price/Felxible-wage Model (SPFWM)

When the nominal wages are flexible, the marginal rate of substitution between consumption and labor is equal to the real wage. Therefore, the wage inflation dynamics are no longer given by (11) but can be recovered from (10). Figure 1 presents impulse responses to a 1% technological shock. Note that unlike in the Ravenna and Walsh (2002) discretionary setup, impulse responses of the output gap and inflation are non-zero under the empirical rule where the central bank targets the output gap and inflation. Also, because wages are flexible, the empirical monetary policy rule is given by (30) with $\gamma_\xi = 0$ (see Table 1 for the other coefficient values).

From the qualitative standpoint, the cost channel does affect the model's dynamics considerably. Inflation deviates further from its steady state and takes longer to return to it, whereas the output gap's impulse response turns from negative to slightly positive before converging to its steady state value. Quantitatively, however, the marginal effect due to the cost channel is very small: the largest differences in impulse responses that arise due to the cost channel are about 0.04%. This is in stark contrast with the large quantitative and qualitative differences that result from adding the nominal wage stickiness assumption to the baseline New Keynesian model, as investigated below.

5.1.2 Flexible-price/Sticky-wage Model (FPSWM)

If prices are assumed to be flexible, the Phillips curve essentially disappears. Instead, the log real marginal cost will be set to zero in all time periods, which allows to define the real wage as:

$$\zeta_t = \bar{\zeta}_t^n + \chi r_t, \tag{31}$$

hence equation (10) no longer needs to be used. Also, assuming that prices are flexible implies that $\gamma_\pi = 0$ in the monetary policy rule (30).

Figure 2 illustrates the impulse responses to a 1% technological shock in this setting. The interest rate impulse response to the technological shock is so small that it cannot significantly widen the real wage gap (see equation (31)). In the absence of price stickiness, this is the only effect that the cost channel adds to the model. As a result, the monetary authority faces little additional difficulty in managing the two variables that enter its objective in this setup: the output gap and wage inflation; hence there is no need for large interest rate decreases. Note also that inflation decreases in response to the shock and quickly reverts to its steady state value, as one would expect in the flexible price setup.

5.1.3 Sticky-price/Sticky-wage Model (SPSWM)

When both nominal wages and prices are assumed to be sticky, the cost channel's additional impact is very small. As Figure 3 illustrates, its presence implies no qualitative or significant quantitative differences in impulse responses. Comparing Figures 1 and 3 one can see that the introduction of sticky nominal wages leads to considerable differences in impulse responses of all variables except output, which is robust to model specification. Relative to nominal wage stickiness, the cost channel adds very little to the New Keynesian model.

This explanation is largely confirmed by the sensitivity analysis performed in the Appendix. The graphs in Appendix F vary parameters θ_w and χ between 0.05 and 0.95 to determine how sensitive the model's variables are to the extent to which the nominal wages are sticky and the cost channel is present. The most striking result is that the value of χ has relatively little impact on the variables' standard deviations; practically all of the variability in the second moments comes from the variation in the value of θ_w. These graphs imply that, in response to a technological shock, the extent of the cost channel matters very little once the assumption of the nominal wage stickiness has been made. The next section, however, highlights the cost channel's important role in response to a monetary policy shock.

5.2 Impulse Responses to a Contractionary Monetary Shock

This section considers augmenting the monetary policy rule given in Table 1 by an exogenous shock whose standard deviation is set to 0.2.[13] Two observations are in order. First, because the technological shock is

[13]The size of this shock is motivated by empirical results. See Kapinos (2004b) for details.

set to zero, the natural level of output is also zero, hence $y = \tilde{y}$. Second, because the empirical literature on the price puzzle studies the effect of a monetary policy shock on the price level, impulse responses for the price and nominal wage levels are also included. I assume that a plausible model has to account for the output-gap/inflation trade-off in its theoretical setup; therefore, only the models with at least two nominal frictions are considered.

Figure 4 illustrates the roles of different frictions in response to a contractionary monetary policy shock. Impulse responses of the nominal interest rate and output seem to be unaffected by the model specification. Differences in the nominal wages and wage inflation appear to be explained by the latter's presence in the monetary policy rule (and, correspondingly, the assumption on the flexibility of the former). That said, the model that assumes flexible wages seems to produce strikingly implausible dynamics as the wage inflation rapidly fluctuates shortly after the shock. The main differences arise in the impulse responses of the price level and inflation. Models with flexible wages clearly do not generate anything even remotely resembling an upward movement in prices found in the data in response to this shock, but neither does the presence of sticky nominal wages alone. The addition of the cost channel accomplishes this task. Rabanal (2003) suggests that the combination of sticky prices and nominal wages produces sticky real wages, which, using (17), introduces inertia into the firm's marginal cost. If either of these two components is flexible, the firm's marginal cost will exhibit considerable flexibility as well. This real wage stickiness channels an increase in interest rates into an increase in the price level. Also interesting is the fact that the duration of the positive response in the price level can match the one found in the data by previous studies: Assuming $\chi = 1$, the duration of the positive response lasts for about 5 quarters, which exceeds most estimates. Assuming a lower value of χ would produce the necessary (shorter) duration of this impulse response.

Figure 5 conducts some basic sensitivity analysis with respect to the changes in some parameters. The magnitude and duration of the 'price puzzle' increase as σ and ϕ increase. Interestingly, as the extent of nominal stickiness in wages and prices increases, the magnitude of the puzzle does not increase, only the duration. Therefore, to generate a sharper positive impulse response of prices to a monetary contraction, one does not need to assume higher degree of nominal stickiness but adjust assumptions about the utility function. A possible explanation for this effect may be that as households become more risk averse, they are more willing to accept higher prices that firms charge due to a monetary contraction, as these households attempt to better smooth consumption over time. Therefore, it takes longer for the negative aggregate demand effect on prices to dominate this positive effect that results from the cost channel.

On the other hand, producing a sharp response of a very short duration, which may be seen in some empirical impulse responses, would necessitate changing the way that nominal stickiness is modelled. In particular, it seems that in a highly inflationary environment the probability of keeping prices constant has to be lower than under circumstances when prices are fairly stable. Future work will address these modelling issues in detail.

6 Alternative Conducts of Monetary Policy

This section investigates the role of endogenous shocks in the comparison between an interest rate monetary policy rule examined above and a money supply rule. The theoretical setup and discussion presented in this section are fairly informal and are meant to provide a starting point for future research.

6.1 Considering a Money Supply Rule

Since the publication of the seminal contribution by Sargent and Wallace (1975) that has argued against interest rate rules and in favor of money supply rules, there has been substantial interest in the literature on the relative advantages of these two ways of conducting monetary policy. Woodford (2003) shows that there may be perfect equivalence between the two alternatives: The central bank's controlling the interest rate and providing adequate money supply to satisfy existing demand for liquidity is tantamount to its controlling the money supply and allowing the interest rate adjust given the money demand. However, there may be important differences in the practical application of these two types of rules.

Section 3 presented several variants of an interest rate rule, in which the central bank changes nominal interest in response to the model's endogenous variables. These rules have been thoroughly discussed in the New Keynesian literature and several forms have been estimated using various datasets. When it comes to monetary policy, however, one frequently hears calls for commitment to fixing the money supply at some exogenously determined level or having it grow at an exogenously determined rate. In either case, in the log-linearized form that describes deviations from the steady state, the value of the money supply would be zero.

There seem to be two practical ways of introducing money into the New Keynesian model. First, one could introduce real money balances into the utility function; this, however, would posit that money enters the second-order approximation of the utility function and, consequently, the central bank's welfare objective. Second, one could introduce money by having a cash-in-advance constraint, which frequently takes the form of the quantity theory of money equation of exchange. Because this alternative avoids the problems with the welfare function, it seems more preferable in this setup.

The general specification of this cash-in-advance constraint used here follows the form:

$$\frac{M_t}{P_t} = \frac{Y_t}{V(Y_t, R_t)}, \tag{32}$$

that is, real money demand depends on output and the velocity of circulation of money, which, in turn, depends on output and nominal interest.[14] After log-linearizing and subtracting the natural levels to get this equation in terms of gaps, we have:

$$\tilde{m}_t - p_t = \eta \tilde{y}_t + \varsigma \tilde{r}_t, \tag{33}$$

where $\eta = 1 - \frac{\partial \log V(\cdot)}{\partial \log Y}$ and $\varsigma = -\frac{\partial \log V(\cdot)}{\partial \log R}$. A money supply policy rule that will be considered below assumes that the monetary authority sets $\tilde{m}_t$ to zero. This allows to recast (33) as an interest rule that targets the current output gap and the price level. This exercise is interesting because, in a model with sticky information, Ball, Mankiw, and Reis (2003) find a version of this rule to be optimal, unlike an interest-rate rule that targets an inflationary term.

[14]This specification of the velocity of circulation allows for a closer comparison with interest rate rules studied in Section 3.

6.2 Estimating the Money Supply Rule

The dataset described in Section 3 lends itself very naturally to the estimation of (33), because over that time the Federal Reserve was widely perceived as controlling the nominal interest rate and letting the money supply adjust freely to meet the demand. To correct for possible persistence in the stochastic shock to (33), I constructed a simple state-space model given by:

$$\tilde{m}_t - p_t = \eta \tilde{y}_t + \varsigma \tilde{r}_t + \mu_t, \tag{34}$$

where μ is the money demand shock, and

$$\mu_t = \rho_\mu \mu_{t-1} + \epsilon_\mu, \tag{35}$$

where ϵ_μ is assumed to be white noise. Estimates of the parameters involved are all significant at the 1% level and are: $\hat{\eta} = 0.584$, $\hat{\varsigma} = -0.009$, $\hat{\rho}_\mu = 0.992$. Note that given these estimates, recasting (33) in terms of an interest rate rule would place a very high weight on the price target and that is roughly two times the size of the weight placed on the output gap target.[15] This implies that one would expect to see more volatility in the output gap term relative to inflation. Also, because the rule targets the price level as opposed to an inflationary term, prices in the model where monetary policy is described by this rule are going to be stationary, which may be criticized as counterfactual. This criticism, however, is of secondary concern here, because the main idea behind this exercise is to evaluate the simulated performance of this rule relative to the one described in Section 3.

6.3 Comparing Money Supply and Interest Rate Rules

Table D in the Appendix presents the second moments generated by the models where an output-gap/inflation tradeoff exists and that are completed by either an interest rate or the money supply rule. The reason why I consider the variance of only inflation, output gap, and wage inflation is because it is only these three variables that may enter the central bank's objective. Two observations hold true regardless of the model

[15] Here, again, the assumption is that the monetary authority sets the money gap term to zero.

specification: the variance of inflation is lower under the money supply rule and the variance of wage inflation is higher. The reason why price targeting leads to lower inflation variability is fairly obvious: targeting a price level as opposed to its rate of change reduces the variability of the price level and, consequently, inflation. Because it can be formally shown (see the appendix on the derivation of the welfare objective) that the greatest weight in the central bank's objective function should be placed on inflation, one can claim, with a fair amount of confidence, that the price targeting rule is superior to a Taylor-type rule.

The primary reason why the wage inflation variance is uniformly higher under the money supply rule is because neither the wage level nor the wage inflation term enters the rule. To overcome this specification problem, one could assume—in a rather ad hoc fashion—that the money circulation velocity depends on, say, wage inflation. This is likely to reduce the variance of wage inflation under price targeting. But going through this exercise may not be necessary if one considers the results from Setup C, where nominal wages are assumed to be flexible and hence wage inflation does not enter the interest rate rule either. In that setup, both price inflation and output gap variance are lower under price targeting than under inflation targeting, hence the money supply rule minimizes the welfare loss.

The reason why applying price targeting may be harder in practice is because, in the other setups, it leads to higher volatility in the output gap, which may, depending on the parametric specification of the welfare objective, outweigh the gains from lower inflation volatility. Hence the policy recommendation on the practical implementation of price targeting needs to be deferred until good estimates of these parameters are obtained.

7 Conclusion

This paper has surveyed the role of the cost channel and nominal wage stickiness in several theoretical environments. The cost channel of transmission of monetary policy stresses that a firm's cost function has the nominal interest rate as one of its arguments. In the New Keynesian model, the cost channel accomplishes two effects: it can generate a tradeoff between the output gap and inflation in response to a technological shock and it can produce a positive response of the price level to a contractionary monetary policy shock.

The role of the cost channel in a model's response to a technological shock hinges on the assumptions made about the behavior of prices and nominal wages. If the prices are flexible, the cost channel hardly alters the model's dynamics. At the same time, if prices and nominal wages are sufficiently sticky, the presence of the cost channel has minimal implications for the model. Therefore, it plays an important role in response to a technological shock only in the setting where prices are sticky and nominal wages are flexible.

The importance of the cost channel in response to a contractionary monetary policy shock is highlighted in the setting with sticky prices and nominal wages. In that model, prices exhibit a positive impulse response found in the data by many researchers and termed the 'price puzzle'. This effect can be achieved because the combination of sticky prices and nominal wages renders the real wages, an important component of the firm's marginal cost function, sticky. Therefore, an increase in the nominal interest rate induces the representative firm to charge higher prices. Over time, however, the negative impact of the higher interest rates on the aggregate demand leads to a reduction in the price level.

Comparison of several estimates of the Federal Reserve's monetary policy rule strongly suggests that it may attempt to affect the labor market by changing nominal interest in response to the expectation of the future wage inflation. This term in the Taylor-type rule is theoretically motivated, as the rule can be rigorously derived as a second-order approximation of the representative household's utility function. The role of this term in monetary policy has not been previously investigated but appears to be important in the exercises reviewed above.

Finally, this paper has questioned whether the Federal Reserve needs to commit to an interest rate rule that targets expected future inflation or whether it should switch to a money supply rule that implies targeting the price level instead. Although price targeting dramatically reduces inflation volatility, in most setups, it also tends to raise the output gap variance. Therefore, considering the practical implementation of price targeting needs to be deferred until precise estimates of the welfare loss parameters can be obtained.

References

[1] Amato, Jeffrey and Thomas Laubach (2000). "Monetary Policy in an Estimated Optimization-based Model with Sticky Prices and Wages." Bank for International Settlements Working Paper No. 83.

[2] Anderson, Gary and George Moore (1985). "A Linear Algebraic Procedure for Solving Linear Perfect Foresight Models". *Economic Letters*, vol. 17, 247-252.

[3] Ball, Laurence, N. Gregory Mankiw, Ricardo Reis (2003). "Monetary Policy for Inattentive Economies." *Mimeo.*

[4] Blanchard, Olivier and Charles M. Kahn (1980). "The Solution of Linear Difference Models under Rational Expectations". *Econometrica*, vol. 48, 1305-1311.

[5] Bonomo, Marco and Carlos Viana de Carvalho (2003). "Endogenous Time-Dependent Rules and Inflation Inertia." *Mimeo.*

[6] Calvo, Guillermo (1983). "Staggered Prices in a Utility Maximizing Framework." *Journal of Monetary Economics*, vol. 12, 383-398.

[7] Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans (2001). "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy". NBER Working Paper 8403.

[8] Clarida, Richard, Jordi Galí, and Mark Gertler (1999). "The Science of Monetary Policy: A New Keynesian Perspective". *Journal of Economic Literature*, vol. 37, 1661-1707.

[9] Cliff, Mike (2003). "GMM and MINZ Program Libraries for MATLAB". *Mimeo.*

[10] Erceg, C.J., D.W. Henderson, and A.T. Levine (2000). "Optimal Monetary Policy with Staggered Wage and Price Contracts". *Journal of Monetary Economics*, vol. 46, 281-313.

[11] Favero, Carlo A. (2001) *Applied Macroeconometrics.* Oxford, UK.

[12] Galí, Jordi (1992). "How Well Does the IS-LM Model Fit Postwar U.S. Data?" *Quaterly Journal of Economics*, vol. 107, 709-738.

[13] Ireland, Peter (2004). "Technology Shocks in the New Keynesian Model". *Mimeo.*

[14] Kapinos, Pavel (2004a). "GMM Estimates of New Keynesian Structural Parameters from the US Data: The Case of Sticky Nominal Wages". *Mimeo.*

[15] Kapinos, Pavel (2004b). "Resolving the Price Puzzle: Theory and Evidence". *Mimeo.*

[16] Rabanal, Pau (2003). "The Cost Channel of Monetary Policy: Further Evidence for the United States and the Euro Area." *IMF Working Paper 03/149.*

[17] Rabanl, Pau and Juan F. Rubio-Ramírez (2003). "Comapring New Keyensian Models of the Business Cycle: A Bayesian Approach". *Mimeo.*

[18] Ravenna, Federico, and Carl E. Walsh (2002). "The Cost Channel in a New Keynesian Model: Evidence and Implications". *Mimeo.*

[19] Sargent, T.J. and N. Wallace (1975). "Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule".*Journal of Political Economy*, vol. 83, 241-254.

[20] Sbordone, Argia (2002). "Prices and Unit Labor Costs: A New Test of Price Stickiness". *Journal of Monetary Economics*, vol. 49, 265-292.

[21] Schmitt-Grohé, Stephanie, and Martín Uribe (2001b). "Optimal Fiscal and Monetary Policy under Sticky Prices". *Mimeo.*

[22] Sims, Christopher A. (1992) "Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy". *European Economic Review*, vol. 36, 975-1000.

[23] Taylor, John B. (1980) "Aggregate Dynamics and Staggered Contracts". *Journal of Political Economy*, vol. 88, 1-23.

[24] Walsh, Carl E. (2000) *Monetary Theory and Policy.* MIT Press, Cambridge, Massachusetts.

[25] Woodford, Michael (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press, Princeton, NJ

[26] Yun, Tack (1996). "Nominal Price Rigidity, Money Supply Endogeniety, and Business Cycles". *Journal of Monetary Economics*, vol. 37, 345-370.

Appendix A: Primitive Parameters of the Model

Table A

Parameter	Value
Discount factor, β	0.99
Elasticity of substitution between goods within each category, ϵ_P	6
Elasticity of substitution between different types of labor, ϵ_W	6
Inverse intertemporal rate of substitution, σ	1.5
Inverse labor supply elasticity, ϕ	0.5
Degree of price stickiness, θ_P	0.75
Autocorrelation coefficient for technological shock, ρ	0.95
Standard deviation of a technological shock, %	1

Appendix B: Optimal Monetary Policy with Sticky Nominal Wages

Following derivations detailed in Ravenna and Walsh (2002) and Erceg et al. (2000), the welfare function, given by the second-order approximation of the household utility function, can be expressed as (omitting time subscripts):

$$\mathbf{W} = -\frac{1}{2}\{(\sigma+\phi)\tilde{y}^2 + \frac{1}{\epsilon_p}\text{var}_f y(f) + (\phi + \frac{1}{\epsilon_w})\text{var}_i n(i)\}$$

Following Woodford (2003) and using the definitions of demand functions for goods and labor, one can express cross-sectional variances in output across the firms and labor supply across the households as:

$$\sum_{t=0}^{\infty}\beta^t \text{var}_f y(f) = \frac{\epsilon_p^2\theta_p}{(1-\theta_p)(1-\theta_p\beta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 + t.i.p,$$

and

$$\sum_{t=0}^{\infty}\beta^t \text{var}_f n(i) = \frac{\epsilon_w^2\theta_w}{(1-\theta_w)(1-\theta_w\beta)}\sum_{t=0}^{\infty}\beta^t\xi_t^2 + t.i.p,$$

where 't.i.p' stands for 'terms independent of (monetary) policy. Therefore, using price inflation term as the numeraire, the welfare function can be expressed as:

$$\sum_{t=0}^{\infty} \beta^t \mathbf{W} = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \delta_{\tilde{y}} \tilde{y}_t^2 + \delta_\xi \xi_t^2 \right],$$

where

$$\delta_{\tilde{y}} = \frac{\lambda_p(\sigma + \phi)}{\epsilon_p},$$

and

$$\delta_\xi = \frac{\lambda_p}{\epsilon_p} \frac{\epsilon_w}{\lambda_w}.$$

Appendix C: Estimation of Monetary Policy Rules

Estimation of performed using Cliff's (2003) GMM package for MATLAB. Two alternative rules with adjustment for persistence were estimated. If the nominal wages are assumed to be sticky the reduced form of the rule is given by:

$$r_t = aE_t\pi_{t+1} + bE_t\xi_{t+1} + c\tilde{y}_t + \rho r_{t-1},$$

where ρ describes the extent of persistence in the policy rule. This is a reduced form version of equation (30) in the text, where the structural parameters can be recovered as follows: $\gamma_\pi = \frac{a}{1-\rho}$, $\gamma_\xi = \frac{b}{1-\rho}$, and $\gamma_{\tilde{y}} = \frac{c}{1-\rho}$. If the nominal wages are assumed to be flexible, the wage inflation term drops out, i.e. $\gamma_\xi = b = 0$.

Using the δ-method, one can calculate the variance $Var(\gamma)$ of the vector of structural parameters as:

$$Var(\gamma) = \nabla\gamma'\Omega\nabla\gamma,$$

where Ω is the covariance matrix of the reduced form estimates and $\nabla\gamma$ is the gradient of a structural parameter with respect to the reduced for parameters.

The results described in the text use quadratically detrended output as the measure of the output gap. Alternative estimates below use H-P filtered output instead.

Table C: Alternative Estimates of the Interest Rate Rule

	Parameters				Test
	γ_π	γ_ξ	$\gamma_{\tilde{y}}$	ρ	J
$\theta_p, \theta_w > 0$	1.636	1.209	1.652	0.748	5.188
	(0.161)	(0.314)	(0.314)	(0.063)	(0.737)
$\theta_w = 0$	2.111		2.629	0.854	8.710
	(0.178)		(0.720)	(0.043)	(0.464)
$\theta_w = 0$		2.079	7.106	0.961	9.293
		(0.988)	(3.077)	(0.017)	(0.411)

These results are qualitatively similar to the ones presented in the text. The only exception is that assuming flexible prices leads to an implausibly high $\gamma_{\tilde{y}}$. Therefore, assuming sticky prices is critical to obtaining plausible results.

Appendix D: Simulation Variances under Alternative Monetary Policy Rules

The interest rate rule (IRR) is described in Section 3 and its specification depends on the assumptions regarding price and nominal wage stickiness. The money supply rule (MSR) is described in Section 5. The variability in the models was generated by a 1% technological shock.

Table D: Simulation Variances—Alternative Conducts of Monetary Policy

	Setup A $\chi = 1,\ \theta_w = 0.75,\ \theta_p = 0.75$		Setup B $\chi = 0,\ \theta_w = 0.75,\ \theta_p = 0.75$		
	MSR	IRR	MSR	IRR	
π	0.0310	0.0846	0.0255	0.0618	
$\tilde{y}$	1.0749	0.0372	0.7288	0.0417	
ξ	0.0492	0.0038	0.0411	0.0028	
	Setup C $\chi = 1,\ \theta_w = 0,\ \theta_p = 0.75$		Setup D $\chi = 1,\ \theta_w = 0.75,\ \theta_p = 0$		
	MSR	IRR	MSR	IRR	
π	0.0002	0.0545	0.3252	1.0505	
$\tilde{y}$	0.0110	0.0406	1.7421	0.0006	
ξ	1.1023	0.3535	0.0415	0.0000	

Appendix E: Theoretical Impulse Responses

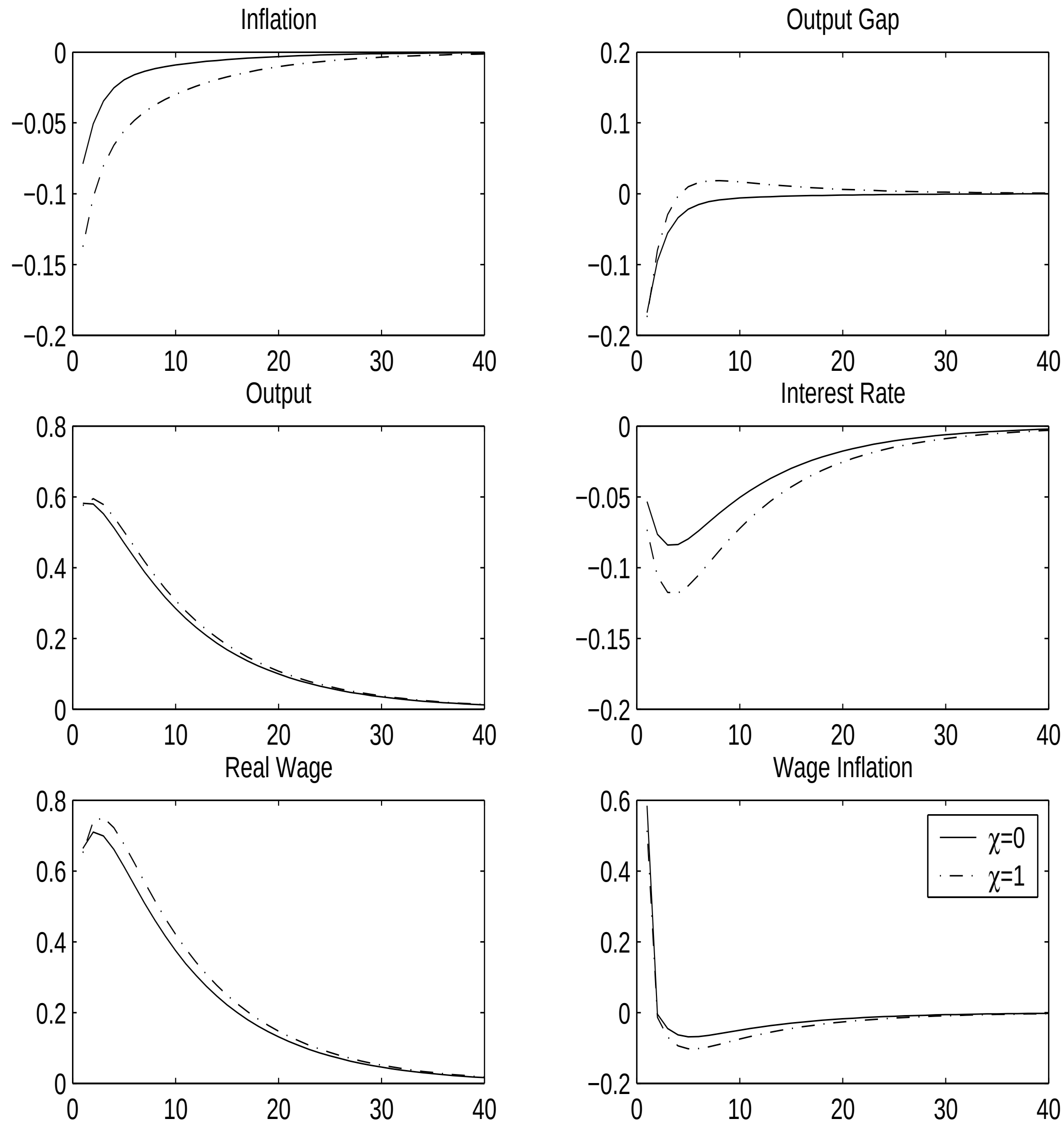

Figure 1: Impulse responses of key variables to 1% technological shock; SPFWM

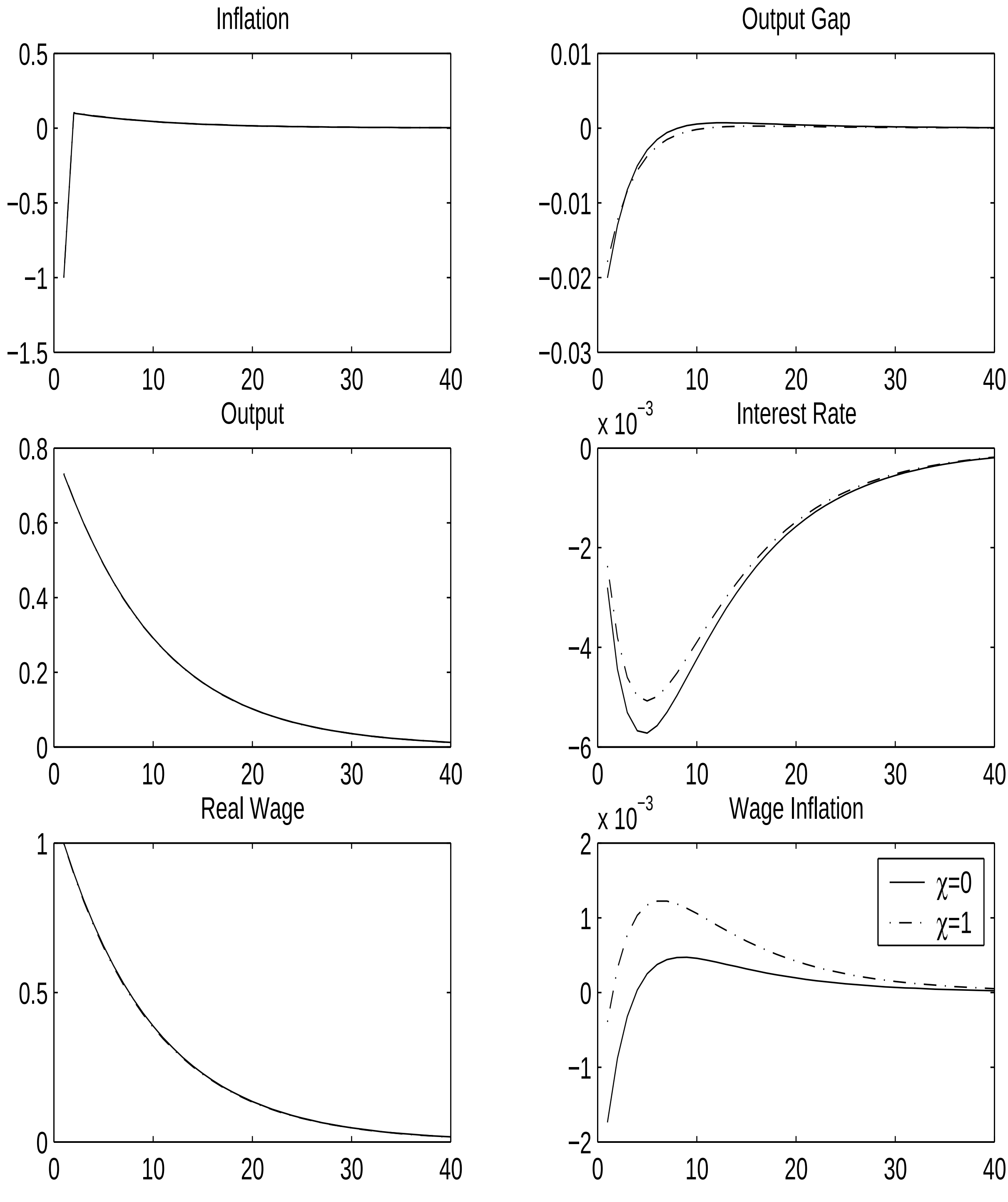

Figure 2: Impulse responses of key variables to 1% technological shock; FPSWM

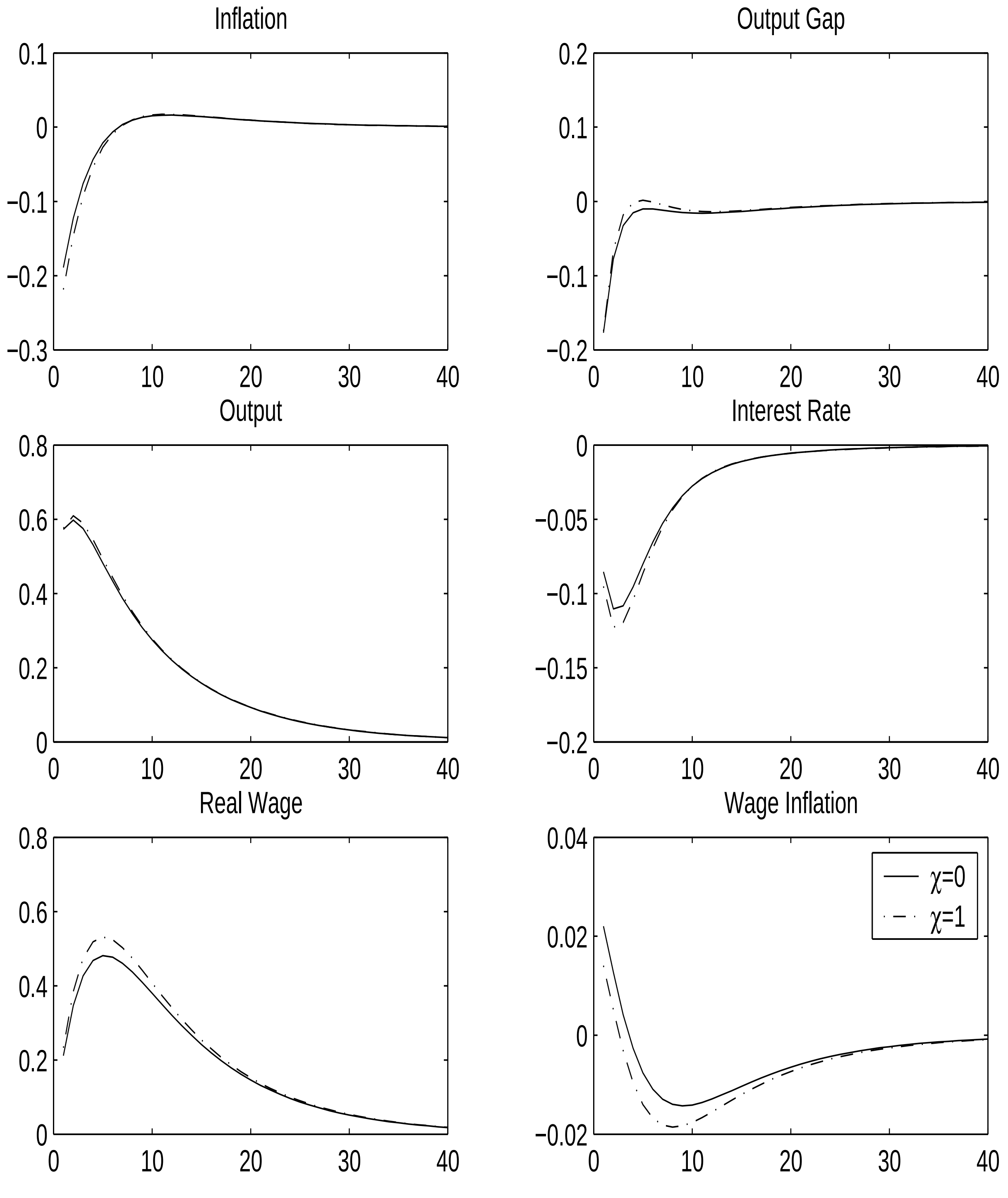

Figure 3: Impulse responses of key variables to 1% technological shock; SPSWM

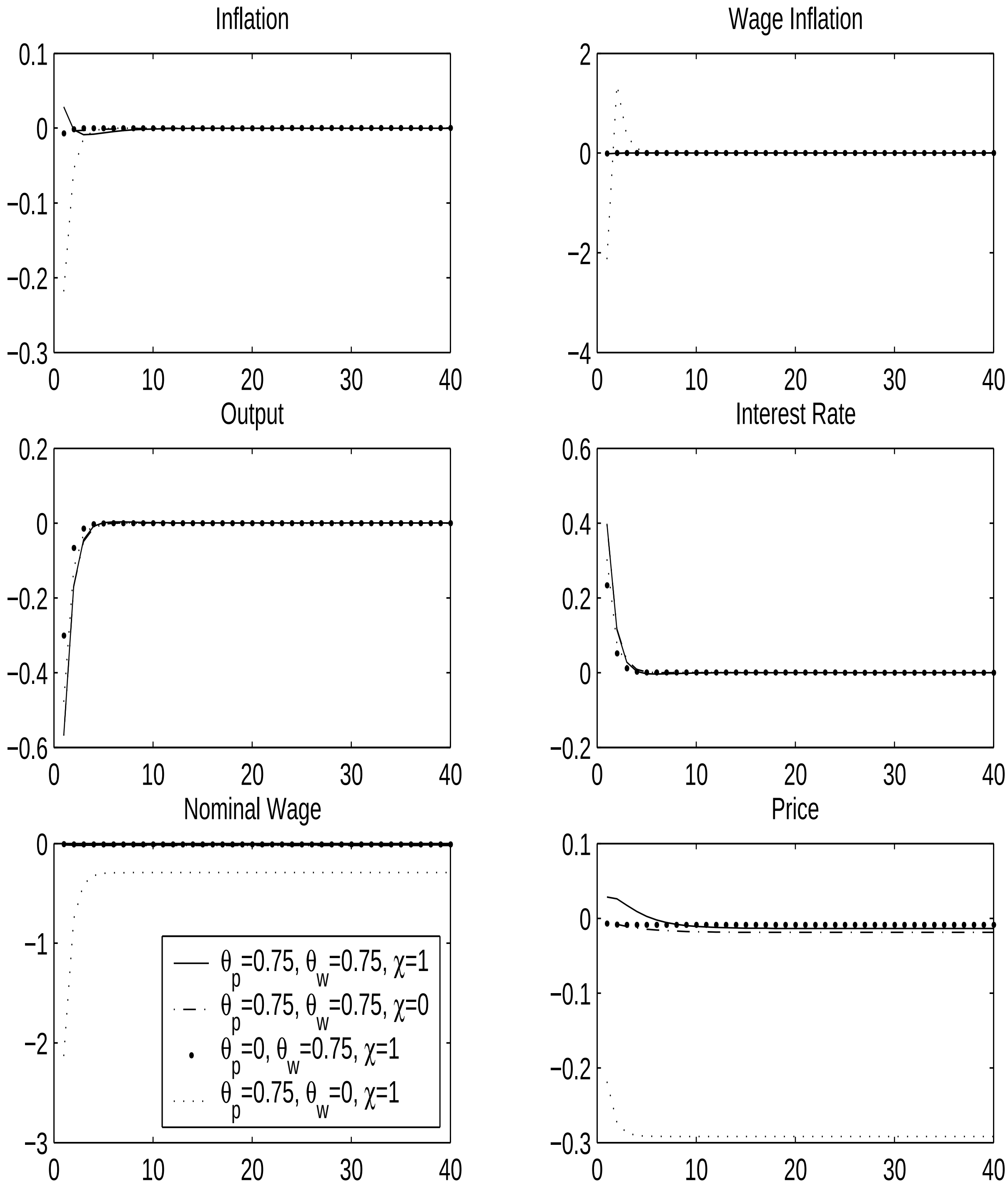

Figure 4: Impulse responses of key variables to a contractionary monetary policy shock

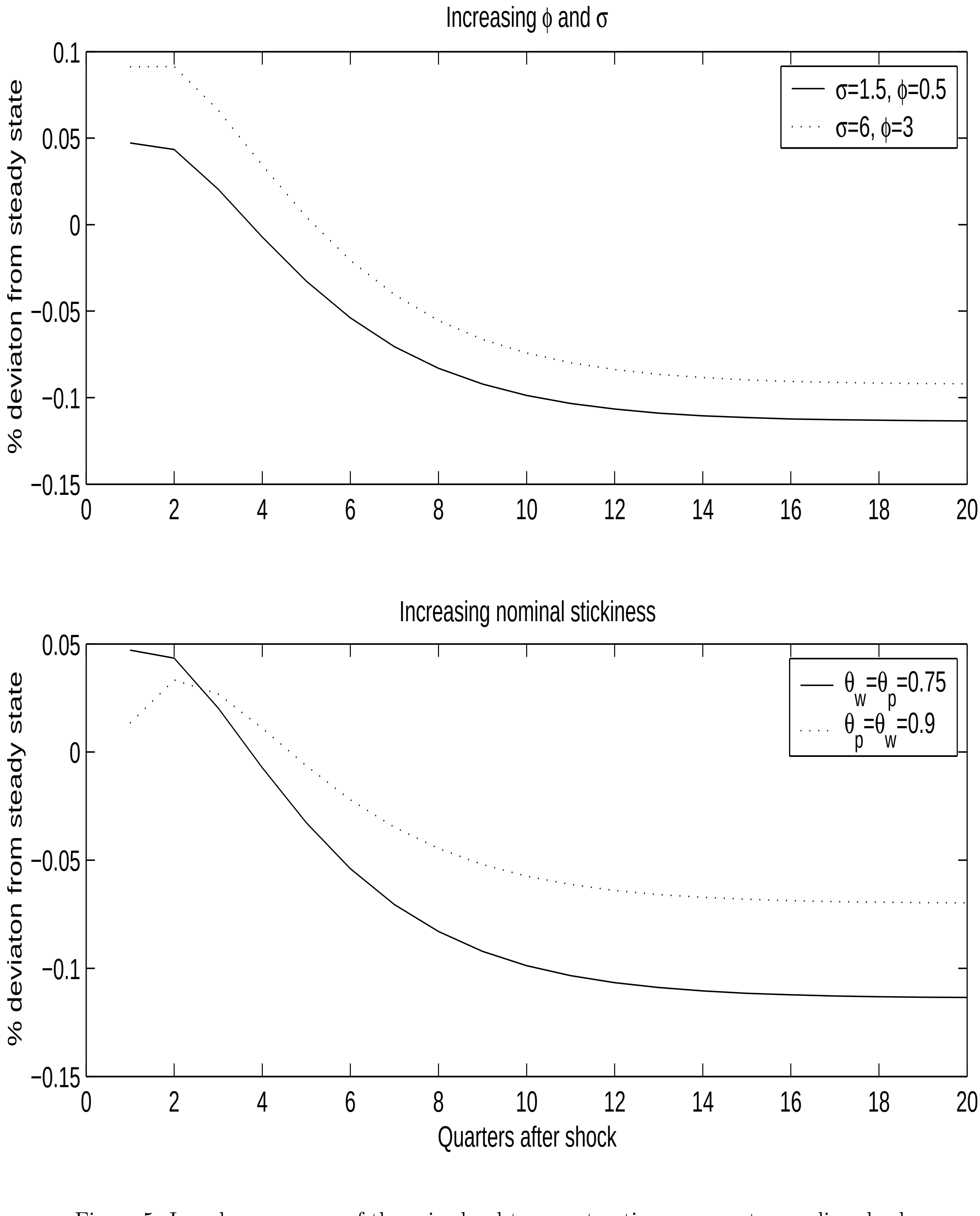

Figure 5: Impulse responses of the price level to a contractionary monetary policy shock

Appendix F: Sensitivity Analysis—Cost Channel vs. Nominal Wage Stickiness

Note that θ_W increases left to right and χ front to back. Both parameters range from 0.05 to 0.95 in steps of 0.05. The monetary policy rule used here is given by: $r_t = (1 - 0.758) * (1.046\tilde{y}_t + 1.481 E_t \pi_{t+1} + 1.095 E_t \xi_{t+1}) + 0.758 r_{t-1}$. The volatility in the output gap depends entirely on the position of the natural level of output relative to the actual level of output. Hence depending on how strong the technological shock is, the volatility of $\tilde{y}$ can either increase in θ_w or decrease or fluctuate as it does under the present calibration.

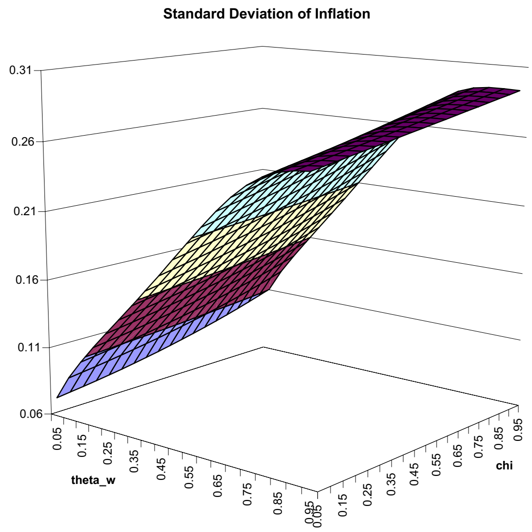

Figure 6: Standard deviations of π in response to 1% technological shock

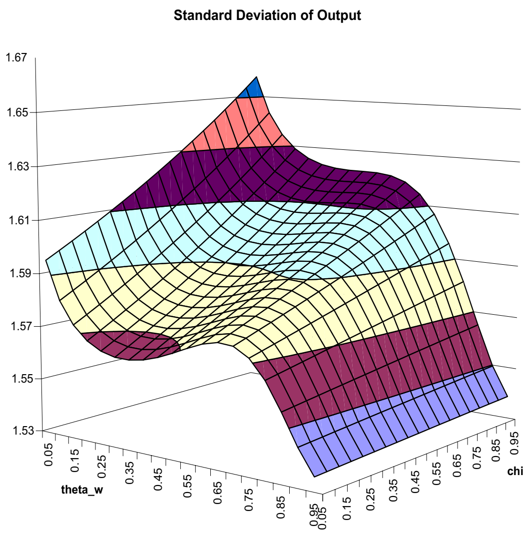

Figure 7: Standard deviations of y in response to 1% technological shock

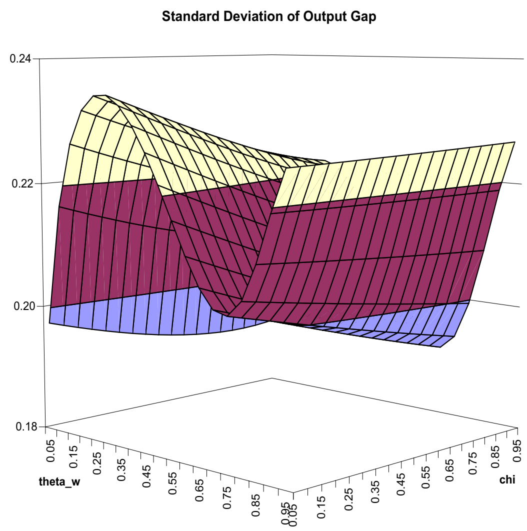

Figure 8: Standard deviations of $\tilde{y}$ in response to 1% technological shock

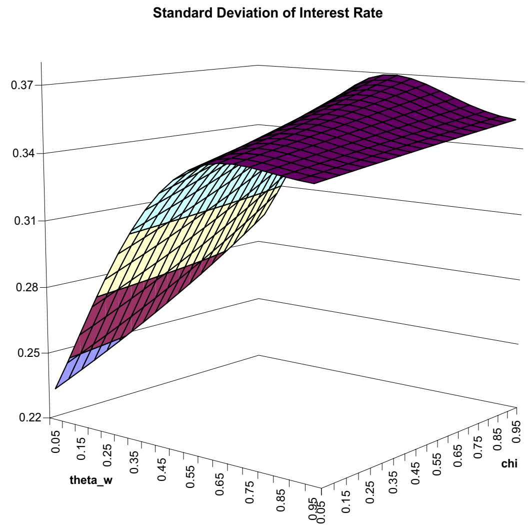

Figure 9: Standard deviations of r in response to 1% technological shock

Figure 10: Standard deviations of ζ in response to 1% technological shock

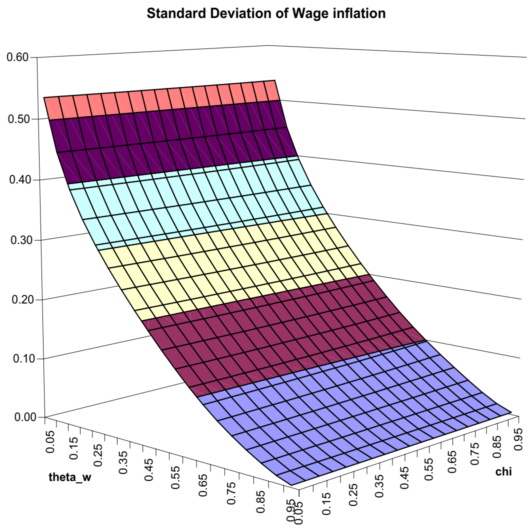

Figure 11: Standard deviations of ξ in response to 1% technological shock