

Knot Theory

Course Description

Take a piece of string, jumble it up, then seal the ends together. The result is a knot. Notice that you can't untie the knot because you've permanently sealed the ends together. We call two knots equivalent if you can move one jumbled piece of string to look exactly like the other without cutting it open. (Using more technical language, knots are embeddings of the circle into 3-space, considered up to ambient isotopy.) Given two knots, can you determine if they are equivalent? If you suspect that they are not equivalent, can you prove it?

Mathematicians have been studying knots ever since the late 1800's when Lord Kelvin incorrectly theorized that all of matter was made up of knotted ether, where different elements corresponded to different knots. Although his theory of matter was incorrect, the study of knots has turned out to be a very rich field of mathematics, specifically a subfield of low-dimensional topology. While knots are the subject of current research by top mathematicians, there are knot theory topics that can be understood and investigated by students as young as middle school.

This course will study knot theory without assuming anything beyond high school mathematics.

Textbook

We will use *The Knot Book* by Colin C. Adams.

Homework

Homework is an important part of any math class. It is important that you practice doing the problems. This will help you to understand the material better and will prepare you for the exams. You are encouraged to discuss the homework, and to work together on the problems. However each student is responsible for the final preparation of his or her own homework papers.

Projects

Two projects will be assigned throughout the semester. Each project will have two parts. The first part will introduce a knot theory concept that is not covered in the book. This part is required. The second part will relate this concept to a question that I don't know the answer to. This part of the project will be worth extra credit and could potentially lead a published math paper.

Exams

There will be one midterm exam and one final exam.

Course Grade Determination

The course grade will be computed as follows:

- Homework - 25%
- Participation - 15%
- Projects - 10%
- Midterm - 25%
- Final - 25%