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Squeezing close to the stability boundaries of the Paul trap

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Abstract

We propose theoretically and prove numerically that an arbitrarily high degree of squeezing is obtained for the wave function of a charged particle in an ideal Paul trap if the trap is operated close to the stability boundaries.

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Since they were first studied theoretically [1,2] and their existence verified experimentally [3], squeezed states [4–6] have found many important applications ranging from spectroscopy [7,8] to gravitational wave detectors [9,10]. Squeezed states also exist in the Paul trap [11], a device developed in the 1950s [12], which since has become an indispensable part of fundamental physics experiments [13,14] and technical applications such as frequency standards [15,16] and quantum computers [17–19]. Squeezed states in the Paul trap have been investigated both theoretically [20–25] and experimentally [26–28]. In all applications the degree of squeezing is important: the more squeezing, the better. In this spirit the purpose of this Letter is to define theoretically and to investigate numerically a scheme with which matter states of arbitrarily high degree of squeezing can be obtained in an ideal Paul trap.

The quantum theory of a single particle in the Paul trap is well known [20–25,29–38]. Focusing on the x

degree of freedom, the uncertainties of position and momentum as a function of scaled time $\tau = \Omega t/2$ (Ω is the rf trap frequency) of a particle of mass m in the trap's ground Floquet state are given by

$$\begin{aligned} \delta x^2(\tau) &= \frac{\hbar}{2m} f(a, q; \tau), \\ \delta p_x^2(\tau) &= \frac{\hbar m}{2} g(a, q; \tau), \end{aligned} \quad (1)$$

where

$$\begin{aligned} f(a, q; \tau) &= \frac{\sigma^2(\tau)}{\bar{\omega}}, \\ g(a, q; \tau) &= \frac{\bar{\omega}}{\sigma^2(\tau)} + \frac{\dot{\sigma}^2(\tau)}{\bar{\omega}} \end{aligned} \quad (2)$$

and (a, q) , $\bar{\omega}$, σ are the trap's control parameters [39], the effective frequency [35] and the classical scaling function [35], respectively. We found that for (a, q) combinations close to the borders of stability of an ideal Paul trap (heavy lines in Fig. 1) arbitrarily high degrees of squeezing can be achieved.

Fig. 2 shows the functions f (dashed line) and g (full line) for $(a = -0.01991329, q = 0.2)$ (marked D

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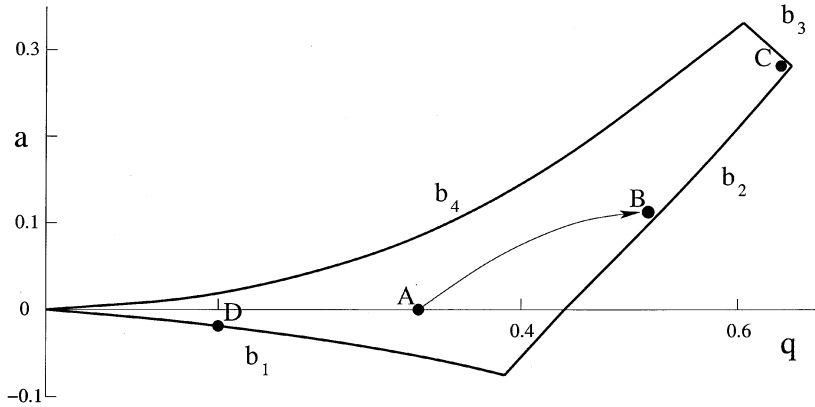


Fig. 1. Schematic illustration of a theoretical scheme for generating squeezed states close to the stability border in an ideal Paul trap. The fundamental stability region is framed by the stability borders (heavy lines) labeled b_1 , b_2 , b_3 and b_4 , respectively. Starting at A the quantum ground state is dragged adiabatically to point B by slowly changing the values of the control parameters a and q . The closer point B is to the border of stability, the larger the degree of squeezing that is achievable with this scheme. Large squeezing is also obtained if, starting at A, the ground state is dragged toward C, D, or any other point close to any of the stability borders b_1, \dots, b_4 .

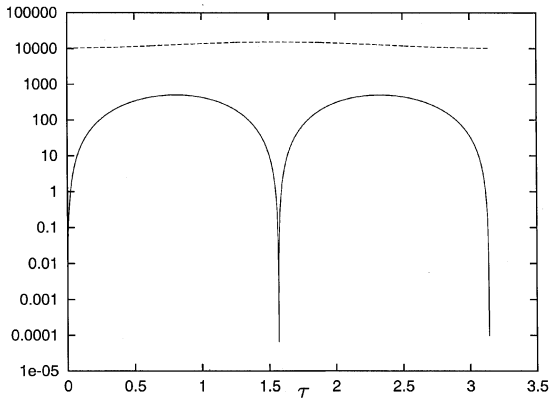


Fig. 2. The functions $f(\tau)$ (dashed line) and $g(\tau)$ (full line), proportional to $\delta x^2(\tau)$ and to $\delta p_x^2(\tau)$, respectively, for parameter combination $(a, q) = (-0.01991329, 0.2)$ very close to the stability border b_1 (point D in Fig. 1) as a function of scaled time τ . f is large, corresponding to anti-squeezing in the x coordinate. At multiples of $\pi/2$, g is smaller than 10^{-4} corresponding to strong squeezing in momentum.

in Fig. 1), a parameter combination very close to the stability border b_1 in Fig. 1. The function f is very large for all τ , corresponding to a very broad distribution in x . The function g , however, reaches values smaller than 10^{-4} at integer multiples of $\pi/2$ corresponding to strong squeezing in momentum.

Since $\bar{\omega} \rightarrow 0$ and $\sigma(\tau)$ stays bounded away from zero as $(a, q) \rightarrow b_1$, the momentum uncertainty δp_x can be made arbitrarily small as $(a, q) \rightarrow b_1$.

While close to b_1 we observe strong squeezing in p_x (and p_y), only very weak squeezing is observed in the z component of the wave function. This leads us to the investigation of a parameter combination close to the intersection of the stability lines b_2 and b_3 (see Fig. 1), $(a = 0.27, q = 0.67)$ (marked C in Fig. 1), where we observe strong squeezing in all three degrees of freedom. But the corner C has even more interesting physics to offer. In contrast with point D where strong squeezing occurs only in p_x , Fig. 3 shows that at C both f and g become very small once every trap cycle, and are $\pi/2$ out of phase. This means that the squeezing ellipse [6] rotates once every trap cycle corresponding to a quantum state that alternates periodically between strong x and p squeezing.

At C all three degrees of freedom, x , y and z , exhibit qualitatively the same squeezing behavior as shown in Fig. 3, with the z components $\pi/2$ out of phase with respect to the x , y components. The closer the operating point of the trap is chosen to the corner formed by b_2 and b_3 , the stronger the squeezing that can be achieved in all three dimensions.

Close to the stability border b_4 large squeezing is also obtained, but this time only for the z coordinate. The detailed squeezing behavior for the z coordinate

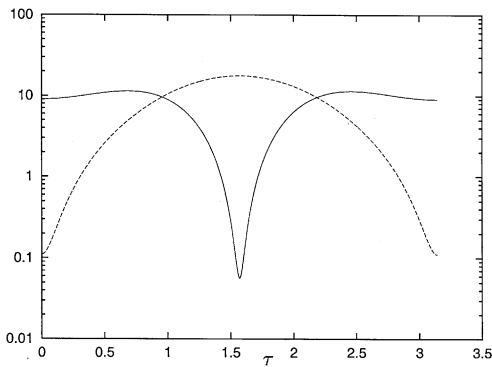


Fig. 3. The functions f (dashed line) and g (full line) (see (2) and the caption of Fig. 2) for the interesting case $(a, q) = (0.27, 0.67)$ (marked C in Fig. 1) at the corner formed by the stability borders b_2 and b_3 . In this case both f and g approach zero very closely once every trap cycle. This means that both x and p_x show periodic squeezing and anti-squeezing. The special operating point $(a, q) = (0.27, 0.67)$ was chosen, since in this case all three degrees of freedom show qualitatively the same periodic squeezing and anti-squeezing behavior.

at b_4 is very similar to the squeezing behavior of the x, y coordinates close to b_1 (see Fig. 2). This is immediately obvious since the stability line b_1 for the radial direction in the Paul trap corresponds to the stability line b_4 for the z direction. Therefore, the physics close to these stability lines is symmetric with respect to the exchange operation $b_1 \leftrightarrow b_4, x, y \leftrightarrow z$.

Squeezed states close to the stability borders are obtained in the following way. Starting in the quantum ground state at point A (see Fig. 1), the particle is dragged [40] toward the border of the stability region by adiabatically changing the a and q parameters. The end point of the resulting trajectory in (a, q) space (indicated by the thin, curved arrow in Fig. 1) is marked by the point B in Fig. 1. According to the above results, the closer the end point B approaches any of the stability borders, the larger the degree of squeezing that is achievable. This is so since the adiabatic dragging procedure with speeds $v_a = \dot{a}$, $v_q = \dot{q}$ transforms the quantum ground state at A into the squeezed quantum ground state at B. To prove this we observe that the single-particle Paul-trap problem is integrable and therefore no Landau–Zener (avoided) crossings [41,42] are encountered on the trajectory from A to B. In this case adiabatic perturbation theory holds which tells us that as long as the conditions

$|v_a| \ll \bar{\omega}$, $|v_q| \ll \bar{\omega}$ are satisfied, transitions to excited states are exponentially small, and the particle ends up in the trap's ground state at B with a probability exponentially close to 1. We confirmed this qualitative argument with detailed numerical calculations.

In summary we defined theoretically and verified numerically a scheme according to which large degrees of squeezing can be achieved in the Paul trap. The degree of squeezing is theoretically unlimited and depends only on the closeness of approach to the stability border of a stability region in (a, q) parameter space of the Paul trap.

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