

INTRODUCTION

This study examines a number of macro variables that represent domestic and foreign influences on bilateral imports to the US from Canada, Japan, and the UK. Through a vector auto regression (VAR) model with short run restrictions, the study is able to determine the importance of shocks from selected macro variables in influencing the fluctuations of bilateral imports to the US over different time horizons.

The study is based off Koray & Lastrapes' paper Real Exchange Rate Volatility on U.S. Bilateral Trade: A Var Approach (1989), which uses the same VAR model to investigate the impact of real exchange rate volatility on U.S. bilateral imports before and after the Bretton Woods system, up until 1985. Due to the availability of bilateral import data, I will be extending their study and only examine data from 1985 to the present day. Furthermore, instead of just focusing on the relationship between exchange rate volatility and bilateral imports, I will be considering shocks from the entire system.

VAR THEORY

Vector Autoregression (VAR) is a procedure for examining how shocks to the variables in a system affect the system over time. The key attribute of the multi-equation approach through VAR is that the variables in the system are completely endogenous. The improvements VAR makes on single equation time-series methods has placed VAR in the limelight of contemporary time-series research.

The VAR model can be easily understood by examining a simple bivariate system with one lag :

- 1.) Consider 2 equations that explain the system of two variables $y_{1,t}$ and $y_{2,t}$. This can be written in vector form as follows...

$$\begin{aligned} y_{1,t} &= \gamma_{10} - \beta_{12}y_{2,t} + \gamma_{11}y_{1,t-1} + \gamma_{12}y_{2,t-1} + \varepsilon_{1,t} \\ y_{2,t} &= \gamma_{20} - \beta_{21}y_{1,t} + \gamma_{21}y_{1,t-1} + \gamma_{22}y_{2,t-1} + \varepsilon_{2,t} \end{aligned}$$

$$\begin{bmatrix} 1 & \beta_{12} \\ \beta_{21} & 1 \end{bmatrix} \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} \gamma_{10} \\ \gamma_{20} \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$$

Where $\varepsilon_{1,t} \sim N(0, \sigma_1^2)$
 $\varepsilon_{2,t} \sim N(0, \sigma_2^2)$

Where $\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \sim N(0, D)$, $D = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$

- 2.) Taking the left product of the inverse matrix of **B** and then by letting $C = B^{-1}\Gamma_0$ and $\phi = B^{-1}\Gamma_1$, the equation is now in a form that an OLS regression can produce parameter estimates.

$$\begin{aligned} Y_t &= B^{-1}\Gamma_0 + B^{-1}\Gamma_1Y_{t-1} + B^{-1}\varepsilon_t \\ (\text{Structural AR model}) \end{aligned} \Rightarrow \begin{aligned} Y_t &= C + \phi Y_{t-1} + e_t \\ (\text{Reduced Form model}) \end{aligned}$$

Where $\begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix} \sim N(0, \Omega)$, $\Omega = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix}$

- 3.) The estimation of the reduced form model yields a total of nine parameters estimates:

$$C_{11}, C_{12}, \phi_{11}, \phi_{12}, \phi_{21}, \phi_{22}, \omega_{11}, \omega_{12}, \omega_{22}$$

However, the original system of equations contains ten parameters:

$$\gamma_{10}, \gamma_{20}, \beta_{12}, \beta_{21}, \gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22}, \sigma_1^2, \sigma_2^2$$

In order to exactly identify the system, econometricians use either Short-Run, Long-Run, or Sign Restrictions.

- 4.) The VAR reduced form model in (2) can be transformed into a moving average (MA) model where each variable in the system is a function of contemporaneous and lagged orthogonal shocks.

$$Y_t = \mu + (I + \phi L + \phi^2 L^2 + \dots + \phi^n L^n) + e_t = \mu + \Psi(L)B^{-1}\varepsilon_t$$

(Wold MA model)

In the Wold MA Form, the VAR results can be analyzed in two ways:

Variance Decomposition shows the proportion of variability of a variable that is attributed to different shocks (ε) in the system over different time horizons.

Impulse Response Functions depict the response of variables to a one standard deviation shock in a specific variable.

VARIABLES

Variables without an * represent U.S. variables while variables with an * represents foreign variables:

- M , M*: M1 money supply
- R , R*: Long Term Government Bond Yield
- P , P*: Consumer Price Index
- Y , Y*: Industrial Production Index
- S: Nominal Exchange Rate
- IMP: Bilateral Imports to the US
- V: Real Exchange Rate Volatility

The VAR identifies orthogonal shocks through the use of short run restrictions, with the ordering:

$$M, R, P, Y, M^*, R^*, P^*, Y^*, V, S, IMP$$

The ordering is determined by the intuition that US, being a large economy, responds more sluggishly to shocks emanating from a smaller country.

REFERENCES

Koray, Faik, and William D. Lastrapes, "Real Exchange Rate Volatility and U.S. Bilateral Trade: A VAR Approach," *Review of Economics and Statistics* 71 (Nov. 1989), 708-712.

RESULTS

point estimate (standard error)	Table 1. - RELATIVE VARIATION IN US IMPORTS EXPLAINED BY INNOVATIONS TO THE SYSTEMS VARIABLES (PERCENT)											
	Shock to:											
System	Time Horizon	M	R	P	Y	M*	R*	P*	Y*	V	S	IMP
Canada	1	0.20 (0.67)	2.64 (2.20)	0.38 (0.97)	0.61 (0.92)	5.39 (2.45)	0.07 (0.65)	1.41 (1.40)	11.33 (3.22)	0.00 (0.35)	0.00 (0.34)	77.96 (4.17)
	12	1.59 (1.99)	4.17 (2.55)	3.86 (3.44)	7.91 (3.87)	5.69 (3.18)	4.15 (3.16)	3.04 (2.20)	11.68 (4.98)	4.28 (3.02)	3.74 (2.66)	49.88 (7.11)
	48	2.16 (2.89)	13.05 (4.40)	3.51 (2.96)	6.68 (3.02)	4.24 (3.52)	25.93 (8.14)	2.91 (2.77)	6.77 (4.70)	5.35 (8.53)	3.70 (3.15)	25.72 (5.91)
Japan	1	0.01 (0.49)	0.02 (0.42)	2.71 (2.10)	0.86 (1.20)	0.52 (0.92)	1.40 (1.23)	0.32 (0.73)	0.19 (0.64)	0.23 (0.49)	0.03 (0.55)	93.71 (3.05)
	12	2.07 (2.39)	10.98 (5.13)	10.02 (5.60)	20.52 (7.28)	2.12 (2.28)	3.60 (3.82)	1.13 (2.15)	3.46 (3.52)	4.33 (3.01)	3.11 (3.23)	38.68 (5.74)
	48	1.29 (4.87)	9.40 (4.80)	10.93 (5.71)	16.07 (6.73)	5.73 (5.72)	3.75 (6.73)	4.01 (3.12)	2.84 (4.85)	8.47 (5.94)	13.85 (8.83)	23.65 (5.61)
United Kingdom	1	4.81 (2.61)	0.04 (0.39)	1.73 (1.54)	0.02 (0.50)	0.14 (0.60)	1.20 (1.14)	0.94 (1.49)	2.96 (2.27)	1.26 (1.13)	0.03 (0.72)	86.86 (3.92)
	12	5.88 (2.85)	5.49 (2.93)	4.30 (2.96)	15.73 (4.63)	3.56 (2.53)	8.15 (3.42)	3.63 (2.23)	5.25 (2.33)	5.91 (2.32)	5.04 (2.57)	37.04 (4.36)
	48	5.13 (4.00)	5.54 (3.33)	12.87 (7.02)	18.26 (5.26)	2.84 (3.18)	8.70 (4.27)	4.07 (2.82)	7.28 (3.68)	5.69 (3.39)	6.75 (3.87)	22.87 (4.76)

Standard errors estimated through Monte Carlo simulation.

Variable abbreviations are M = money supply, R = interest rate, P = price level, Y = output, V = real exchange rate volatility, S = nominal exchange rate
 IMP = real US bilateral imports. * denotes foreign variables (US is the domestic country)

- In accordance with Koray and Lastrapes' results, shocks to real exchange rate volatility explains little of the variability in bilateral imports (no more than 8.5% in any of the three systems).
- All three systems show that on impact, import shocks explain the most variability in imports. As we expand the time horizon, import shocks become less important. The exception to this is Canada, where import shocks appear to be much more persistent.
- Consistent with economic theory, shocks to domestic income also has a significant impact on import fluctuations., , particularly over the longer time horizon. Once again, Canada is the exception, imports from Canada do not appear to respond much to U.S. income shock..
- A possible explanation for the difference between Canada and the other two systems is that the U.S. may import a greater percentage of staple goods from Canada than from Japan or U.K., which tends to be more income inelastic.
- This study can be extended by considering a structural model with long run restrictions, which would attribute more economic meaning to the shocks and a basis for more analysis.

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