Systems of Numerical Acquisition in Cognitive Development

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Introduction

There two core cognitive systems that support numerical acquisition in children:

•**Parallel individuation** allows one to process up to four objects simultaneously

•The **Analog Magnitude** system allows one to distinguish between set sizes as a function of their ratio.
- •Follows **scalar variability**: the error of an estimate increases linearly with respect to increasing set size (the coefficient of variation stays constant).[1]

Knower-levels are a categorization of children by numerical ability.[2]
•**Subset-knowers** have mapped the words "one" to "four" onto conceptual representations of those discrete quantities.
•**CP-knowers** have learned the **Cardinal Principle**.
- •When counting a set, the last number used is the size of the set.
- •Adding one object to a set means there is always a different, specific term to describe the larger amount.

Study

A task called **fast cards** is used to gauge mapping between verbal numbers and these conceptual systems
•Children are shown sets of objects (1, 2, 3, 4, 6, 8, 10, or 14) in quick succession and are prompted to guess the amount without counting.

The relationship between this "**target**" number and the accuracy of their **estimate** should help evaluate the respective roles of both systems in development.

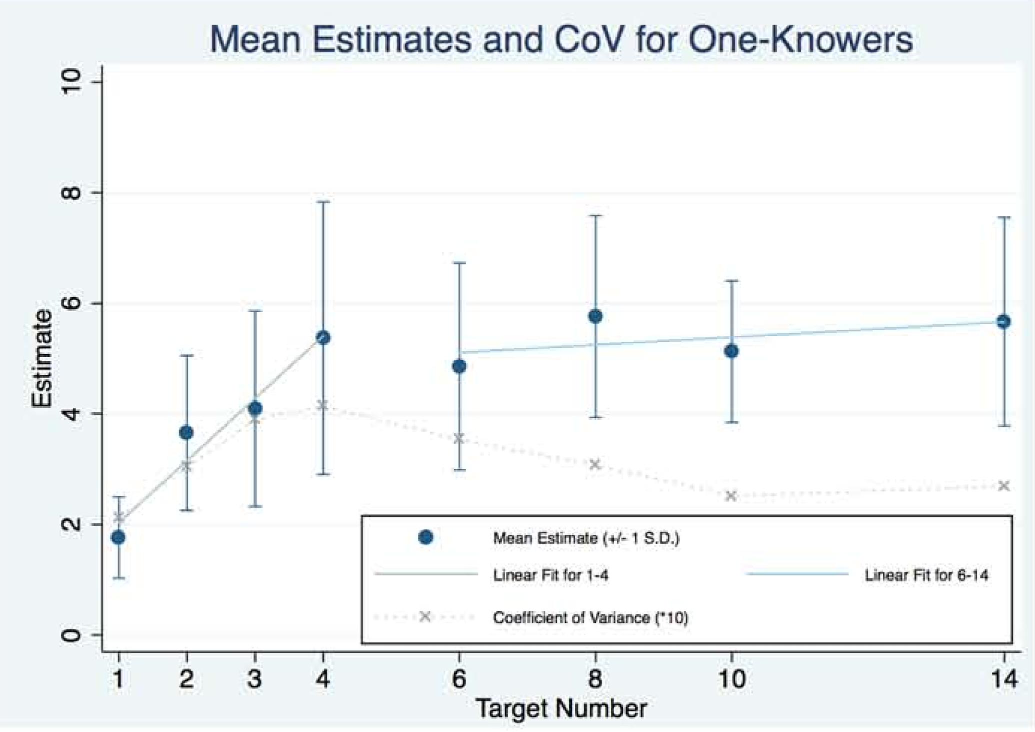
Research question: When in number acquisition do analog magnitudes come into play?
•Signatures of the analog magnitude system are increasing means and a constant coefficient of variation.
•Thus, they will appear if the analog magnitude system is invoked.[3]

Analysis

•Meta-analysis of 7 different fast-cards studies consisting of 306 subjects
•Assessment of mapping strength between analog magnitudes and verbal count list shows:
- • Larger estimates for larger numbers
- • Scalar variability for estimates over 4.
- •Improvement with knower-level

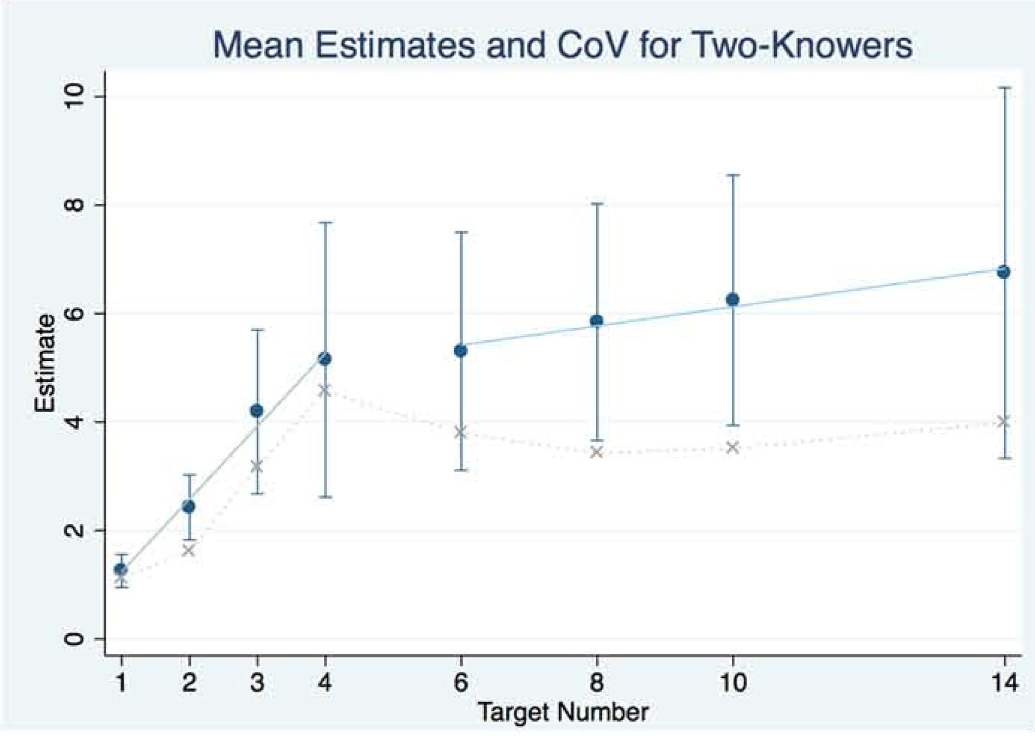
•Note: Significantly positive slopes with a value of 1 indicate means that estimates and target number increase at the same rate.
•Constant CoV indicates scalar variability.

One-Knowers



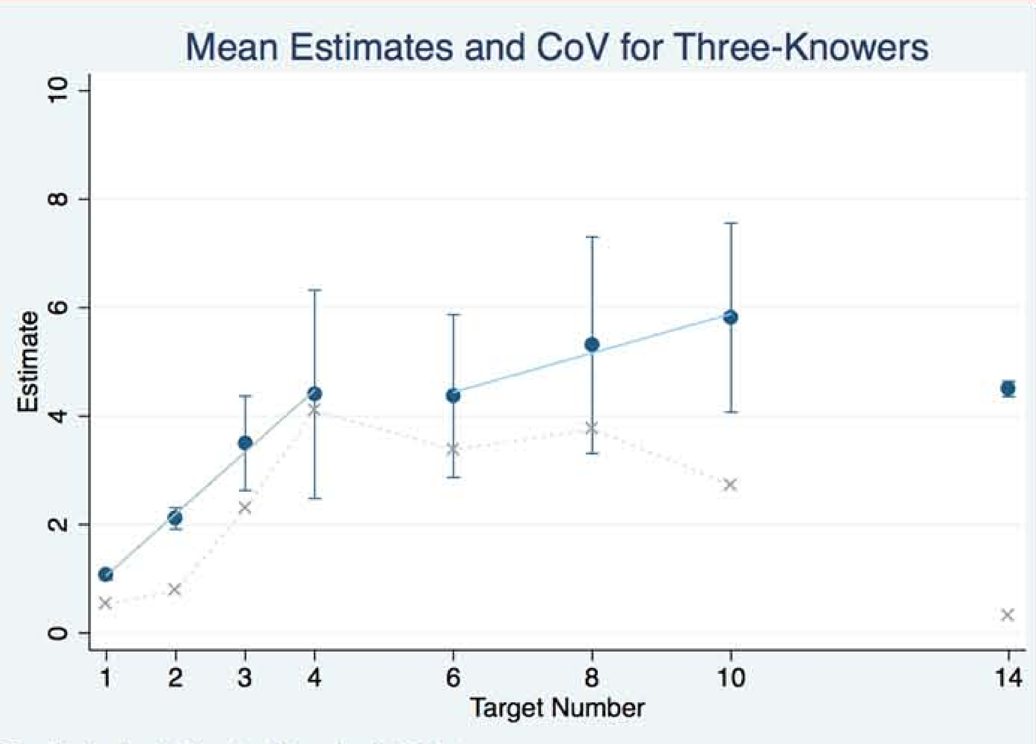
•**1-4 Slope** represents excellent accuracy; it is positive and 1. Estimates are very accurate within the subset, even discounting 1 (β=1.19*, p= 0). Variation is high.
•**6-14 Slope** is not significant. (β=0.092, p=0.472)
•**6-14 CoV** is constant.

Two-Knowers



•**1-4 Slope** is very positive, also near 1. (β=1.33*, p=0). Variation is lower for 1 and 2.
•**6-14 Slope** is significant; it demonstrates larger estimates for larger target numbers. (β=0.232*, p=0.020)
•**6-14 CoV** is constant, i.e. near flat.

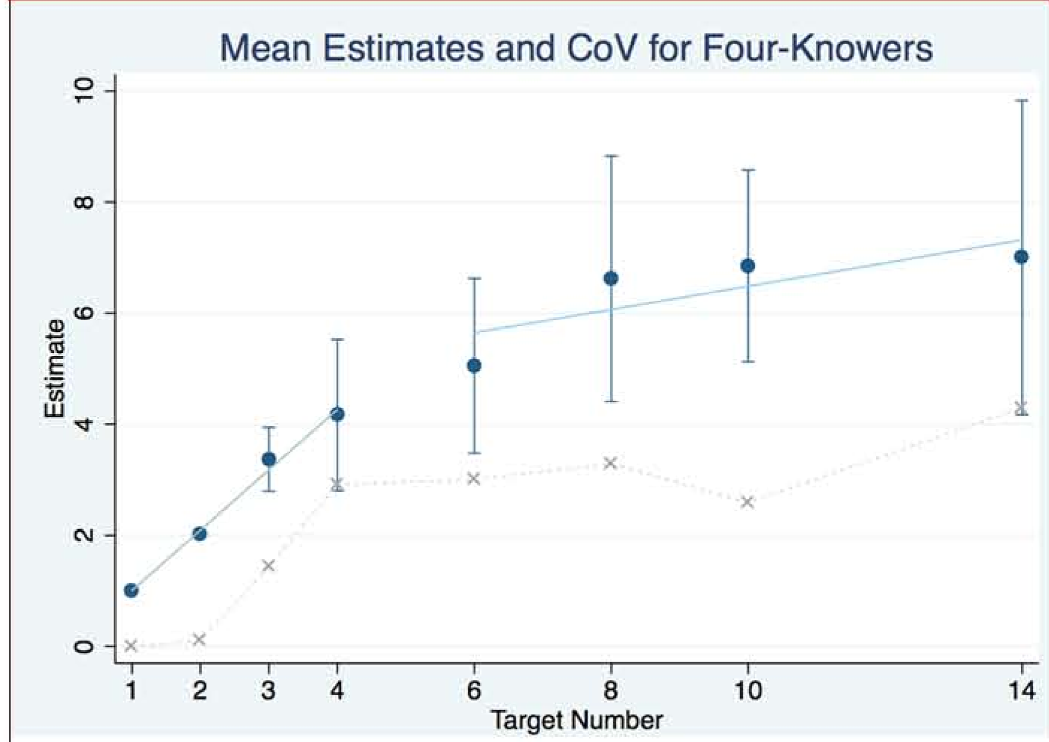
Three-Knowers



Target Number 14 excluded, number of observations insignificant

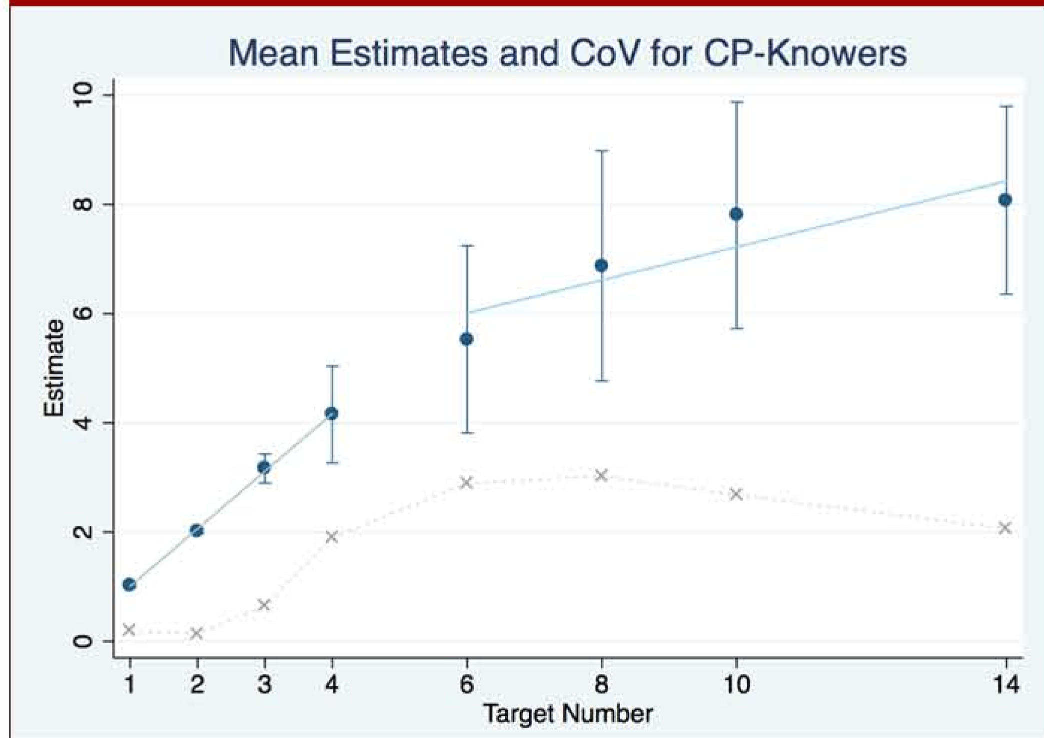
•**1-4 Slope** is positive. Es (β=1.14*,p=0) Variation for numbers and under 3 is very low.
•**6-10 Slope** shows mean estimates increasing with target number. (β=.361*, p=.002)
•**6-10 CoV** is fairly constant.

Four-Knowers



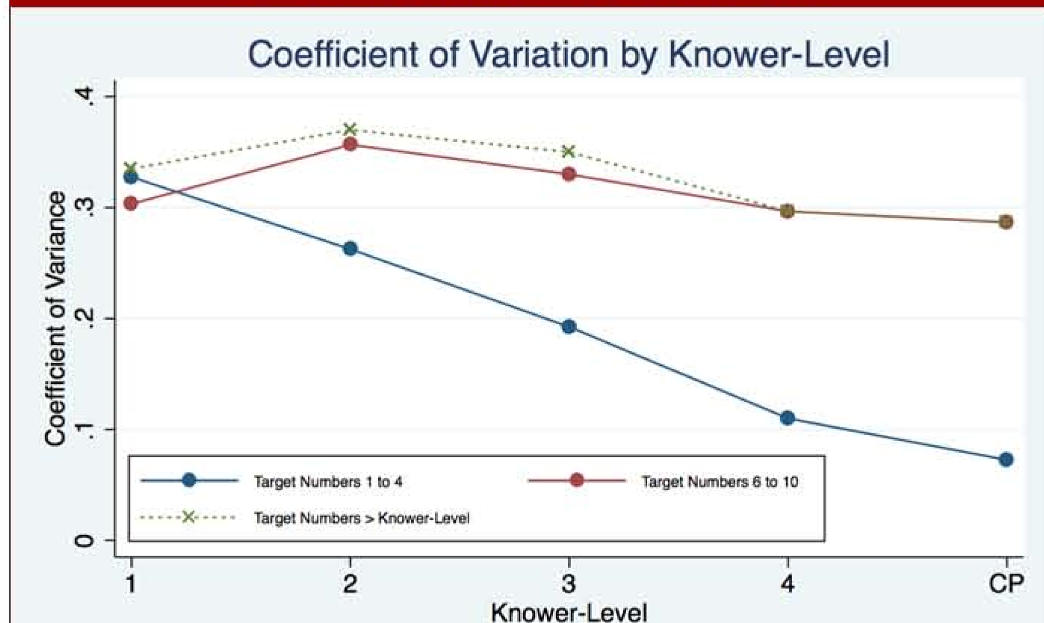
•**1-4 Slope** is posiive and very close to 1. (β=1.09*, p=0). Varation is very low for numbers below 4.
•**6-14 Slope** again shows that mean estimates are increasing linearly. (β=0.303*, p=0.002)
•**6-14 CoV** is constant

CP-Knowers



•**1-4 Slope** is positive (β=1.05*, p=0). Estimates increase at the same rate as target numbers. Variation is the lowest for 1-4.
•**6-14 Slope** shows estimates and target numbers increasing at close rate. (β=0.408*, p=0)
•**6-14 CoV** is constant.

Coefficient of Variation



•A near-constant CoV for numbers 6-10 suggests the influence of analog magnitudes.
- •Accuracy of numbers directly outside knower-level follows a similar pattern.

•Decreasing CoV for target numbers 1-4 fits expectations from the "give-n" task
- •Thus, number production seems to work in the same way as number comprehension.

Summary and Conclusions

Linear Regression Coefficients for Estimate and CoV

	1-4		6-10	
N-Level	β_{tn} on Est.	β_{tn} on CoV	B_{tn} on Est.	B_{tn} on CoV
1	1.19	.091*	.092	-.007
2	1.25*	.120*	.232*	-.003
3	1.14*	.121*	.399*†	-.005
4	1.09*	.106*	.303*	-.003
5	1.05*	.055*	.408*	-.005

*Significant, p<0.05
†6-10 is used for 3-knowers due to lack of data for 14
Linear regression model with the dependent variable as either **Estimate** or **CoV** with independent variables as **Target Number**, **Age**, and **Sex**.

The coefficients above describe the effect of target number on estimate (left) and CoV (right); an increase in target number by 1 will have the above effects on these two dependent variables. As seen in the above table, target number has no effect on CoV for 6-14. Therefore, scalar variability holds for these conditions.

Even for numbers outside their knower-level, children's estimates remain accurate. This accuracy is decreases with set size, however, for numbers greater than 4. As knower-level increases, so does the 6-10 slope (β=.32*, p=0).

Parallel individuation appears to be the primary influence for targets less than 4. 1-4 Slopes are close to 1 for all subset-knowers, but lower knower-levels have more variation.
•All 1-4 slopes approach 1; they stop overestimating as knower-level increases (β=-0.178*, p=0) and their y-intercepts appear to decrease.
•Becoming a CP-knower marks the most substantial leap in numerical ability (β=1.46*, p=0).

All evidence points to the existence of mappings between analog magnitudes and set sizes before the acquisition of the Cardinal Principle.

References

[1]Dehaene, S. (2001) Précis of the number sense. *Mind & Language, 16*, 16—36.
[2]Wynn, K. (1992) Children's Acquisition of the Number Words and the Counting System. *Cognitive Psychology, 24*, 220-251.
[3]Le Corre, M & Carey, S. (2007) One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition 105*. 395-438

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